

# Strong Electroweak Symmetry Breaking Generating Masses Dynamically

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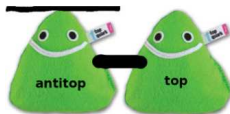
June 28, 2012



# Composite Higgs particle

## Higgs mechanism

- Electroweak symmetry breaking: how particles gain mass
- No scalar found in nature
- Higgs could be composite



## Generating Masses Dynamically

- No mass terms
- Through interactions

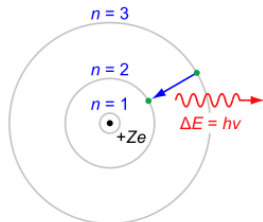
## Strong

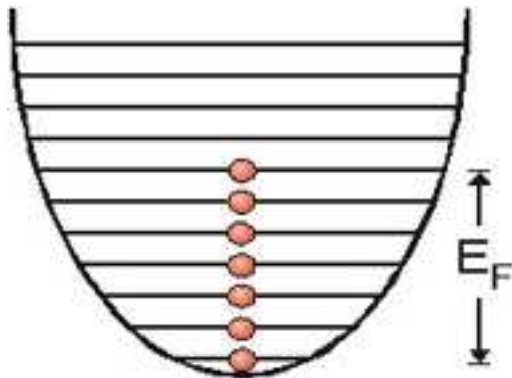
Constituent quark masses in QCD

- 1 Condensed Matter: BCS Theory
- 2 Particle Physics: Nambu–Jona-Lasinio Model
- 3 Custodial Symmetry
- 4 Theories with Strong Dynamics
- 5 Phenomenology

# Old idea: Cooper pairs

Bardeen, Cooper, Schrieffer (1957)





## Cooper pairs

+2 electrons

→ Cooper: weak attraction gives bound state  $< 2\varepsilon_F$

## The Condensate-Inducing Interaction

$$\mathcal{L} = \sum_{\alpha=\uparrow,\downarrow} \phi_{\alpha}^{*}(\mathbf{x}, \tau) \left\{ \hbar \frac{\partial}{\partial \tau} - \frac{\hbar^2 \nabla^2}{2m_{\alpha}} - \mu_{\alpha} \right\} \phi_{\alpha}(\mathbf{x}, \tau) + V_0 \phi_{\uparrow}^{*}(\mathbf{x}, \tau) \phi_{\downarrow}^{*}(\mathbf{x}, \tau) \phi_{\downarrow}(\mathbf{x}, \tau) \phi_{\uparrow}(\mathbf{x}, \tau)$$

Approximate

## Pairing Field

$$\langle \Delta(\mathbf{x}, \tau) \rangle = V_0 \langle \phi_{\downarrow}(\mathbf{x}, \tau) \phi_{\uparrow}(\mathbf{x}, \tau) \rangle .$$

Approximate

## Pairing Field Fluctuation

$$\Delta(\mathbf{x}, \tau) = \Delta + \Delta'(\mathbf{x}, \tau)$$

- $\Delta = \text{minimum}$
- $\rightarrow$  linear terms vanish.
- $\rightarrow \mathcal{L}_{\text{eff}}(\Delta'^*, \Delta', \Delta^*, \Delta)$

# The Gap

## Propagator pole

$$\mathbf{G}_{\Delta}(\mathbf{k}, i\omega_n) = \frac{-\hbar}{-(\hbar\omega_n)^2 - [(\varepsilon_{\mathbf{k}} - \mu)^2 + |\Delta|^2]} \begin{bmatrix} -i\hbar\omega_n - (\varepsilon_{\mathbf{k}} - \mu) & \Delta \\ \Delta^* & -i\hbar\omega_n + \varepsilon_{\mathbf{k}} - \mu \end{bmatrix}.$$

$$\hbar\omega = \sqrt{(\varepsilon_{\mathbf{k}} - \mu)^2 + |\Delta|^2} \equiv \hbar\omega_{\mathbf{k}}$$

Minimum of pole (when  $\varepsilon_{\mathbf{k}}^2 = \frac{\hbar^2 k^2}{2m} = \mu) = |\Delta|$



→ Break Cooper pair with  $2\Delta = \text{gap}$ .



Linear terms vanish:

## Gap Equation

In Fourier Space:

$$\Delta = \frac{V_0}{\hbar\beta V} \sum_{\mathbf{k},n} -\frac{\hbar\Delta}{(\hbar\omega_n)^2 + (\varepsilon_{\mathbf{k}} - \mu)^2 + |\Delta|^2}.$$

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## Requirements for composite Higgs

- Four-fermion interaction
- No bare masses
- Gap Equation for binding energy

## Higgs Mechanism

- Nambu-Goldstone bosons
- Gauge Boson Masses

## Approximation

Ignore light  $u, d, s, c$  quarks

## Four-fermion interaction

$$\mathcal{L} = i\bar{\psi}\not{D}\psi + G_t(\bar{\psi}_L t_R)(\bar{t}_R \psi_L).$$

$$\psi = \begin{bmatrix} t \\ b \end{bmatrix}$$

$$\not{D} = D^\mu \gamma_\mu = \left[ \partial^\mu + i\frac{g_s}{2} G_a^\mu L_a + i\frac{g}{2} W_b^\mu \sigma_b + i\frac{g'}{2} B^\mu Y \right] \gamma_\mu.$$

## Rewrite Lagrangian:

$$\mathcal{L}'_0 = \bar{t}i\gamma^\mu \partial_\mu t + \bar{b}i\gamma^\mu \partial_\mu b - m_t \bar{t}t$$

$$\mathcal{L}'_1 = m_t \bar{t}t + G_t(\bar{\psi}_L t_R)(\bar{t}_R \psi_L).$$

# Dynamical Mass Generation

- Start without bare mass:  $m_0 = 0$ ;
- Insert physical mass in propagator:  $\frac{1}{\not{p} - m_P - \Sigma} \rightarrow$  demand  $\Sigma = 0$ ;

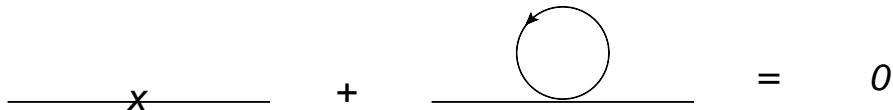
## Interactions

$$\mathcal{L}'_I = m_t \bar{t}t + G_t (\bar{\psi}_L t_R) (\bar{t}_R \psi_L).$$

$$\begin{aligned} -i \Sigma_t = & \text{---} \times \text{---} + \text{---} \bigcirc \text{---} + \text{---} \bigcirc \times \text{---} + \text{---} \bigcirc \bigcirc \text{---} \\ & + \text{---} \bigcirc \bigcirc \bigcirc \text{---} + \text{---} \bigcirc \bigcirc \times \text{---} + \text{---} \bigcirc \times \bigcirc \text{---} + \dots \end{aligned}$$

Figure: From Cristian Valenzuela Roubillard (thesis, 2005)

# Gap Equation for Top Mass


$$\text{---} \times \text{---} + \text{---} \text{---} \text{---} = 0$$

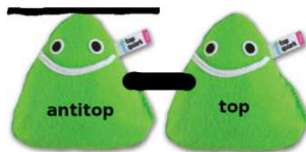
## Gap Equation for NJL-model

Leading order  $N_c$ :

$$\begin{aligned} m_t &= -\frac{1}{2} G_t \langle \bar{t}t \rangle \\ &= 2G_t N_c m_t \frac{i}{(2\pi)^4} \int d^4l \frac{1}{l^2 - m_t^2}; \end{aligned}$$

## Gap Equation in BCS theory

$$\Delta = \frac{V_0}{\hbar\beta V} \sum_{\mathbf{k}, n} -\frac{\hbar\Delta}{(\hbar\omega_n)^2 + (\varepsilon_{\mathbf{k}} - \mu)^2 + |\Delta|^2}.$$

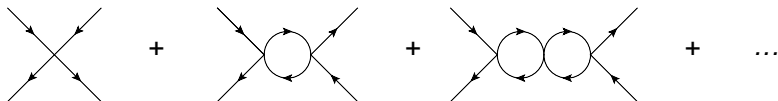


Gap equation:

$$m_t = m_t G_t \frac{N_c}{8\pi^2} \left[ \Lambda^2 - m_t^2 \log \frac{\Lambda^2}{m_t^2} \right]$$

Valid for positive log:  $G_t \geq \frac{8\pi^2}{N_c \Lambda^2} \equiv G_{\text{crit}}$ .

# Composite Higgs State



## Scalar channel

$$\Gamma_s(p^2) = \int d^4x e^{ipx} \langle 0 | T[\bar{t}t](x) [\bar{t}t](0) | 0 \rangle$$



## Scalar channel

$$\begin{aligned}\Gamma_s(p^2) &= \int d^4x e^{ipx} \langle 0 | T[\bar{t}t](x) [\bar{t}t](0) | 0 \rangle \\ &= \frac{-1}{2N_c} \left\{ (4m_t^2 - p^2) \frac{1}{16\pi^2} \int_0^1 dx \log \left( \frac{\Lambda^2}{m_t^2 - x(1-x)p^2} \right) \right\}^{-1}\end{aligned}$$

## Pole of Scalar Channel:

Scalar bound state with mass  $2m_t$ .

## Integral with surface terms

$$\int \frac{d^4l}{(2\pi)^4} \frac{1}{(p+l)^2 - m_t^2} = \int \frac{d^4l}{(2\pi)^4} \frac{1}{l^2 - m_t^2} - \frac{ip^2}{32\pi^2},$$

# Other Bound States

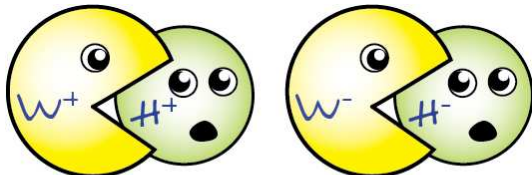
## Four channels in interaction

$$G_t(\bar{\psi}_L t_R)(\bar{t}_R \psi_L) = \frac{1}{4} G_t [(\bar{t}t)(\bar{t}t) + (\bar{t}i\gamma_5 t)(\bar{t}i\gamma_5 t) + (\bar{b}(1 + \gamma_5)t)(\bar{t}(1 - \gamma_5)b)],$$

## Nambu-Goldstone Bosons

- Pseudoscalar Channel:  $\langle T[\bar{t}i\gamma_5 t](x)[\bar{t}(-i)\gamma_5 t](0) \rangle$
- Charged Channels:  $\langle T[\bar{t}(1 - \gamma_5)b](x)[\bar{b}(1 + \gamma_5)t](0) \rangle$ , h.c.

All have poles at  $p^2 = 0 \rightarrow$  **Massless Goldstone Bosons.**

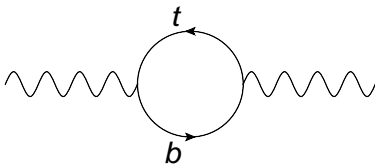


# Mass of the W boson

$$\longrightarrow + \longrightarrow \times \longrightarrow + \longrightarrow \times \longrightarrow \times \longrightarrow + \dots = \Longrightarrow$$

## Corrections from mass interaction to propagator

$$\frac{1}{\not{p}} + \frac{1}{\not{p}} m \frac{1}{\not{p}} + \frac{1}{\not{p}} m \frac{1}{\not{p}} m \frac{1}{\not{p}} + \dots = \frac{1}{\not{p}} \frac{1}{1 - \frac{m}{\not{p}}} = \frac{1}{\not{p} - m}$$



## Renormalized W propagator

$$\frac{-i(g_{\mu\nu} - p_\mu p_\nu / p^2)}{p^2} \left( \frac{1}{1 - \Pi_W(p^2)} \right)$$

# Mass of the W boson



$$[\Pi_W^{\mu\nu}]_2 = \frac{ig}{\sqrt{2}} N_c \int \frac{d^4 l}{(2\pi)^4} (-1) \text{Tr} \left[ \frac{i}{l + \not{p}} \gamma^\mu \frac{i}{l - m_t} \right] i\Gamma_c(p^2) \\ \times \frac{ig}{\sqrt{2}} N_c \int \frac{d^4 l}{(2\pi)^4} (-1) \text{Tr} \left[ \frac{i}{l + \not{p}} \gamma^\nu \frac{i}{l - m_t} \right]$$

## Massless Goldstone Bosons

W boson has eaten NGB's

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## Global Custodial Symmetry

- Must exist:  $\rightarrow$  Relationship between W and Z boson masses
- Model independent

## Peskin-Takeuchi Parameters

- Oblique corrections (vacuum polarization diagrams)
- Contain custodial symmetry
- Constrain new theories

## $\rho$ parameter in SM

$$\frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = \frac{M_W^2 (g^2 + (g')^2)}{g^2 M_Z^2} \equiv \rho$$

- SM: global  $SO(4) = SU(2)_L \otimes SU(2)_R$  symmetry
- Broken by fermion masses to  $SU(2) \equiv$  custodial symmetry
- Custodial symmetry ensures  $\rho \simeq 1$

## Higgs Matrix

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^0 & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}$$

$$\mathcal{L}_{Higgs} = \frac{1}{2} \text{Tr}[(D_\mu \Phi)^\dagger D^\mu \Phi] + \mu^2 \text{Tr}[\Phi^\dagger \Phi] - \lambda \text{Tr}[\Phi^\dagger \Phi \Phi^\dagger \Phi]$$

$$D_\mu \Phi = \partial_\mu \Phi + \frac{1}{2} ig W_\mu^a \tau_a \Phi - \frac{1}{2} ig' B_\mu \Phi \tau_3.$$

- Global  $SO(4)$  Broken explicitly by  $\frac{1}{2} ig' B_\mu \Phi \tau_3$  to  $SU(2) \equiv$  custodial symmetry
- $\rightarrow$  Higgs scalar is no special case of global  $SU(2)$



## Custodial symmetry holds

Find similar  $\rho$ -parameter



Figure: *Dance of the gauge bosons in vacuum* by Dr. Regina Valluzzi

# W and Z masses: at $p^2 = M^2$

## Pole of propagator: self-energy

$$\frac{-i(g_{\mu\nu} - p_\mu p_\nu / p^2)}{p^2} \left( \frac{1}{1 - \Pi_W(p^2)} \right)$$
$$\frac{1 - \Pi_W(p^2)}{(g')^2} = \frac{1}{(\bar{g}'(p^2))^2} - \frac{\bar{f}^2(p^2)}{p^2}.$$

## $\rho$ parameter

$$\frac{M_W^2}{M_Z^2} \left( \frac{g^2(M_Z^2) + g'^2(M_Z^2)}{(\bar{g}')^2(M_Z^2)} \right) = \frac{\bar{f}^2(M_W^2)}{f^2(M_Z^2)}.$$

## SM $\rho$ parameter

$$\frac{M_W^2(g^2 + (g')^2)}{g^2 M_Z^2} = \rho$$

## $\rho$ -parameter in $T$

$$\rho = 1 + \delta\rho_{SM} + \alpha T.$$

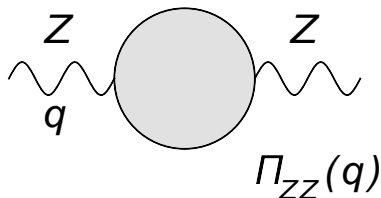
## LEP, SLAC measurements

$$S = -0.16 \pm 0.14 \quad (-0.10)$$

$$T = -0.21 \pm 0.16 \quad (+0.10)$$

$$U = 0.25 \pm 0.24 \quad (+0.01)$$

# Peskin-Takeuchi parameters



## S-, T-, and U-parameters

$$\alpha S = 4s_w^2 c_w^2 \left[ \Pi'_{ZZ}(0) - \frac{c_w^2 - s_w^2}{s_w c_w} \Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0) \right]$$

$$\alpha T = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}$$

$$\alpha U = 4s_w^2 \left[ \Pi'_{WW}(0) - c_w^2 \Pi'_{ZZ}(0) - 2s_w c_w \Pi'_{Z\gamma}(0) - s_w^2 \Pi'_{\gamma\gamma}(0) \right]$$

# Overview

- 1 Condensed Matter: BCS Theory
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## Properties

- New gauge groups added to SM
- Symmetry broken by  $\Phi$  and / or composite Higgs
- SM Gauge boson masses through interactions

## Examples

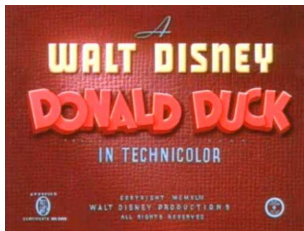
- Topcolor
- Technicolor
- Topcolor-assisted technicolor
- Top Seesaw

## The Gauge Group

$$SU(3)_{1/2} \otimes SU(3)_3 \otimes U(1)_{1/2} \otimes U(1)_3 \otimes SU(2)_L$$
$$\xrightarrow{\langle H_t \rangle = f_{\pi_t}} SU(3)_{QCD} \otimes U(1)_{EM}$$

## Problems

- Top too light for EWSB
- $N_c$  limit
- UV-completion (above  $\Lambda$ ) unknown
- Top Higgs  $\langle \bar{t}t \rangle \equiv H_t < 300$  GeV excluded if top-pion  $\geq 150$  GeV.



Example: Minimal model (Susskind, Weinberg)

$$SU(N_T) \times SU(3) \times SU(2)_L \times U(1)_Y$$

## Problems

- Multiple generations (Extended Technicolor)
- Electroweak Precision Measurements (Walking Technicolor)
- Heavy quark masses (Walking Technicolor)





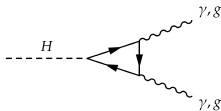
## Viable theories

- Topcolor-assisted Technicolor (TC2)
- Top Seesaw: add  $\chi$  (extended:  $\omega$ )

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## Top Higgs

- Stronger coupling to top than SM Higgs
- Expect  $m_{H_t} \leq 2m_t = 350 \text{ GeV}$
- $m_{H_t} < 300 \text{ GeV}$  excluded if  $m_{\pi_t} > 150 \text{ GeV}$



## Forward-Backward Asymmetry

$$A_{FB}^f = \frac{N_F^f - N_B^f}{N_F^f + N_B^f}$$

CDF (Tevatron): predicted  $0.030 \pm 0.007$  for  $\bar{t}t + \text{bg}$ ;  
measured  $A_{FB}^t = 0.210 \pm 0.049$

## TC2

$$SU(3)_{1/2} \otimes SU(3)_3 \otimes U(1)_{1/2} \otimes U(1)_3 \xrightarrow{\langle \Phi \rangle} SU(3)_{QCD} \otimes U(1)_Y$$

$$SU(2)_L \otimes U(1)_Y \xrightarrow{\langle H \rangle = f_T, \langle H_t \rangle = f_{\pi_t}} U(1)_{EM}$$

## Combined VEV

$$v_{ew}^2 = f_T^2 + f_{\pi_t}^2$$

## Auxiliary Fields

$$\mathcal{L} = i\bar{\psi}\not{D}\psi + c\hbar\bar{\psi}\psi - m^2\hbar^2 \rightarrow i\bar{\psi}\not{D}\psi + c^2(\bar{\psi}\psi)^2$$

## For NJL model:

$$\bar{\psi}i\not{D}\psi + G_t\bar{\psi}_L t_R \bar{t}_R \psi_L \rightarrow \bar{\psi}i\not{D}\psi - m_{t0}^2 H_t^\dagger H_t + g_{t0}(\bar{\psi}_L t_R H_t + \text{h.c.})$$

## Techni-fermion condensate

Similar auxiliary field  $H$

Yukawa e.g.  $Y_{uC} C_L H u_R$ .

## Linear Sigma Model

$$H_t = \exp\left(\frac{i\pi_t^a \tau_a}{\sqrt{2}f_{\pi_t}}\right) \begin{pmatrix} f_{\pi_t} + \frac{1}{\sqrt{2}}h_t \\ 0 \end{pmatrix}$$

$$\mathcal{L}_\Sigma = g_{t0}(\bar{\psi}_L t_R H_t + \text{h.c.}) \approx \frac{\lambda_t}{\sqrt{2}}(f_{\pi_t} \bar{t}_L t_R + \bar{t}_L i\pi_t^0 t_R + \bar{t}_L h_t t_R) + \text{h.c.} + \dots$$

$$M_U = f_T \begin{pmatrix} Y_{uU} & Y_{uC} & \sim 0 \\ Y_{cU} & Y_{cC} & \sim 0 \\ c_{Tu} \frac{M}{\Lambda} & c_{Tc} \frac{M}{\Lambda} & + \frac{m_t}{f_T} \end{pmatrix}$$



## Quark mixing: Diagonalize Mass Matrix

$$t_R \rightarrow V_{11}^{Ru} t_R + V_{21}^{Ru} c_R + V_{31}^{Ru} u_R.$$

## Effective Lagrangian

$$\mathcal{L}_{A_{FB}^t} = g_{tu\pi_t} i\pi_t^0 \bar{t}_L u_R + g_{tuh_t} h_t \bar{t}_L u_R + g_{tu\rho} \rho_\mu \bar{t}_R \gamma^\mu u_R + h.c.$$

# Forward-Backward Asymmetry of the Top

$$\mathcal{L}_{A_{FB}^t} = g_{t0}(\bar{\psi}_L t_R H_t + \text{h.c.}) \rightarrow g_{\pi_t} i\pi_t^0 \bar{t}_L u_R + g_{tuh_t} h_t \bar{t}_L u_R + g_{\rho} \rho_\mu \bar{t}_R \gamma^\mu u_R + \text{h.c.}$$

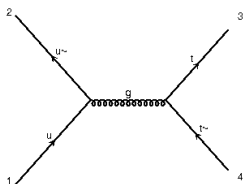


diagram 1

EFTHT=0, EFTPIT=0, EFRHO=0, QCD=2, QED=0

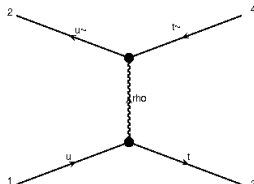


diagram 2

EFTHT=0, EFTPIT=0, EFRHC

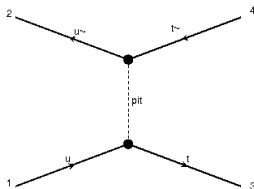


diagram 3

EFTHT=0, EFTPIT=2, EFRHO=0, QCD=0, QED=0

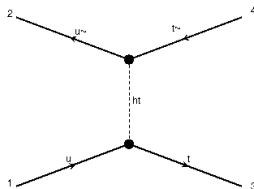


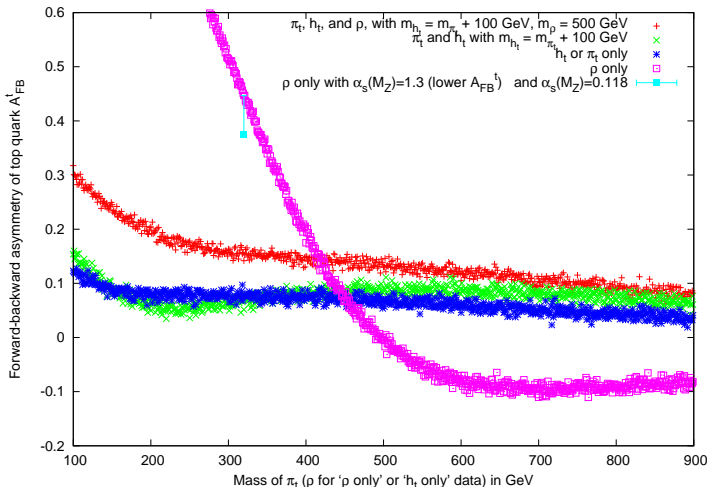
diagram 4

EFTHT=2, EFTPIT=0, EFRHC

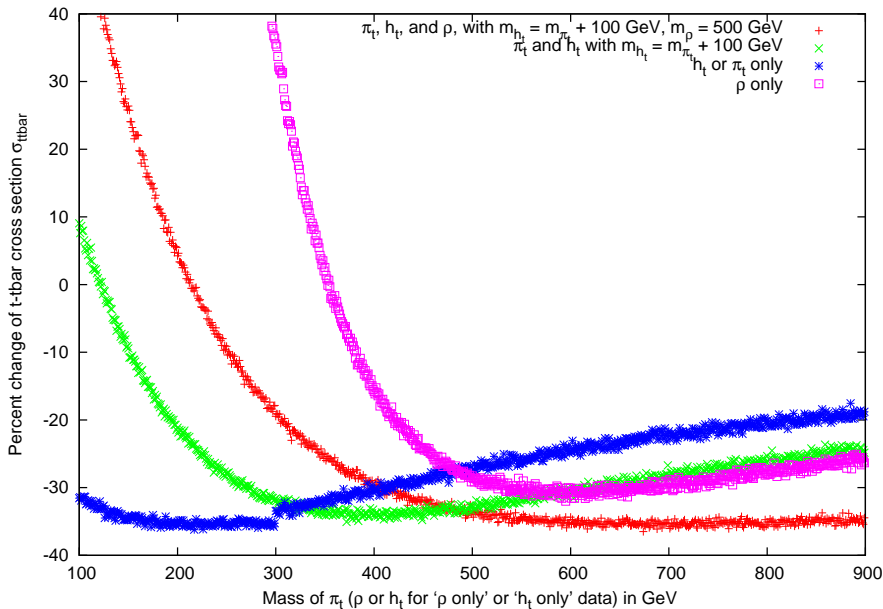


# $A_{FB}^t$ - Forward-Backward Asymmetry of the Top

CDF (Tevatron): predicted  $0.030 \pm 0.007$  for  $\bar{t}t + bg$ ;  
measured  $A_{FB}^t = 0.210 \pm 0.049$

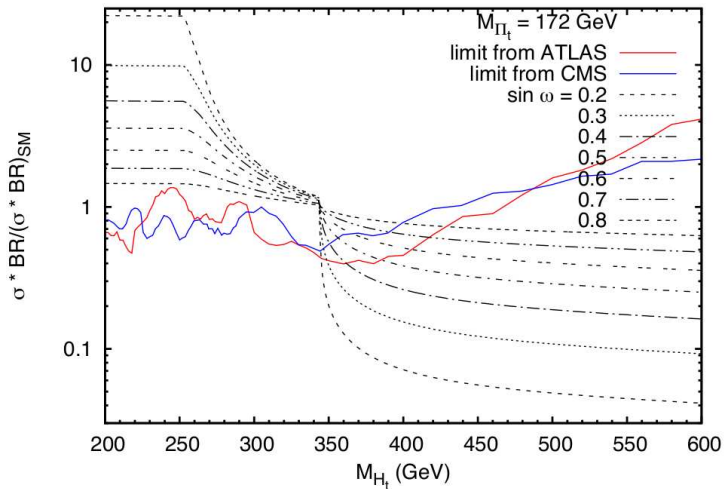


# $\Delta\sigma_{t\bar{t}}$ - Deviation of cross section



# WW branching ratio

Chivukula, Simmons, Coleppa, Logan, Martin (2011) arXiv:1108.4000



## Condensates can break electroweak symmetry:

- BCS theory: Cooper pairs give photon mass
- NJL model: Quark condensate gives gauge bosons mass
- Goldstone bosons and particle of mass  $2m_t$  appear
- Custodial symmetry could still hold
- TC2 still viable
- $A_{FB}^t$  could be explained by TC2

# Discussion

- Custodial symmetry in NJL model by defining parameters
- $A_{FB}^t$  could be explained by other models
- $\rho$  Lagrangian added by hand
- What is  $\Phi$  in TC2?
- TC2 increasingly constrained
- ...

