

Strong Electroweak Symmetry Breaking Generating Masses Dynamically

Supervisor: Prof. Eric Laenen

Jory Sonneveld

Nikhef / Utrecht University

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1 BCS Theory

2 Nambu-Jona-Lasinio Model

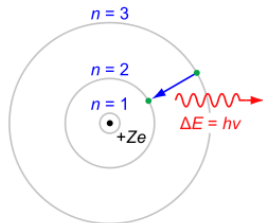
3 Custodial Symmetry

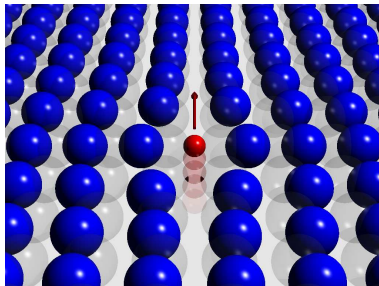
4 Conclusion

5 Outlook

Old idea: Cooper pairs

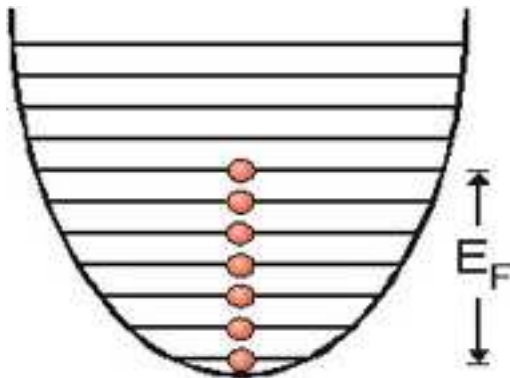
Bardeen, Cooper, Schrieffer (1957)





Phonon: Scattering mediators

photon:light wave = phonon:sound wave



Cooper pairs

+2 electrons

→ Cooper: weak attraction gives bound state $< 2\varepsilon_F$

Four-fermion Short-Ranged Interaction

$$\mathcal{L} = \sum_{\alpha=\uparrow,\downarrow} \phi_{\alpha}^{*}(\mathbf{x}, \tau) \left\{ \hbar \frac{\partial}{\partial \tau} - \frac{\hbar^2 \nabla^2}{2m_{\alpha}} - \mu_{\alpha} \right\} \phi_{\alpha}(\mathbf{x}, \tau) + V_0 \phi_{\uparrow}^{*}(\mathbf{x}, \tau) \phi_{\downarrow}^{*}(\mathbf{x}, \tau) \phi_{\downarrow}(\mathbf{x}, \tau) \phi_{\uparrow}(\mathbf{x}, \tau)$$

Approximate

Pairing Field

$$\langle \Delta(\mathbf{x}, \tau) \rangle = V_0 \langle \phi_{\downarrow}(\mathbf{x}, \tau) \phi_{\uparrow}(\mathbf{x}, \tau) \rangle .$$

Approximate

Pairing Field Fluctuation

$$\Delta(\mathbf{x}, \tau) = \Delta + \Delta'(\mathbf{x}, \tau)$$

- $\Delta = \text{minimum}$
- \rightarrow linear terms vanish.
- $\rightarrow \mathcal{L}_{\text{eff}}(\Delta'^*, \Delta', \Delta^*, \Delta)$

The Gap

Propagator pole

$$\mathbf{G}_{\Delta}(\mathbf{k}, i\omega_n) = \frac{-\hbar}{-(\hbar\omega_n)^2 - [(\varepsilon_{\mathbf{k}} - \mu)^2 + |\Delta|^2]} \begin{bmatrix} -i\hbar\omega_n - (\varepsilon_{\mathbf{k}} - \mu) & \Delta \\ \Delta^* & -i\hbar\omega_n + \varepsilon_{\mathbf{k}} - \mu \end{bmatrix}.$$

$$\hbar\omega = \sqrt{(\varepsilon_{\mathbf{k}} - \mu)^2 + |\Delta|^2} \equiv \hbar\omega_{\mathbf{k}}$$

Minimum of pole (when $\varepsilon_k^2 = \frac{\hbar^2 k^2}{2m} = \mu) = |\Delta|$



→ Break Cooper pair with $2\Delta = \text{gap}$.

Gap Equation

Linear terms vanish:

Gap Equation

In Fourier Space:

$$\Delta = \frac{V_0}{\hbar\beta V} \sum_{\mathbf{k},n} -\frac{\hbar\Delta}{(\hbar\omega_n)^2 + (\varepsilon_{\mathbf{k}} - \mu)^2 + |\Delta|^2}.$$

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Four-fermion interaction

$$\mathcal{L} = \bar{\psi} i \not{D} \psi + G_t (\bar{\psi}_L t_R) (\bar{t}_R \psi_L).$$

$$\psi = \begin{bmatrix} t \\ b \end{bmatrix}$$

$$\not{D} = D^\mu \gamma_\mu = \left[\partial^\mu + i \frac{g_s}{2} G_a^\mu L_a + i \frac{g}{2} W_b^\mu \sigma_b + i \frac{g'}{2} B^\mu Y \right] \gamma_\mu.$$

Dynamical Mass Generation

- Start without bare mass: $m_0 = 0$;
- Insert physical mass in propagator: $\frac{1}{\not{p} - m_P - \Sigma} \rightarrow$ demand $\Sigma = 0$.

$$-i \Sigma_t = \text{---} \times \text{---} + \text{---} \bigcirc \text{---} + \text{---} \bigcirc \times \text{---} + \text{---} \bigcirc \bigcirc \text{---} \\ + \text{---} \bigcirc \bigcirc \bigcirc \text{---} + \text{---} \bigcirc \bigcirc \times \text{---} + \text{---} \bigcirc \times \bigcirc \text{---} + \dots$$

Figure: From Cristian Valenzuela Roubillard (thesis, 2005)

Gap Equation



A Feynman diagram equation. On the left, a horizontal line with an 'x' in the middle. To its right is a plus sign. Further right is another horizontal line with a circular loop on top, with an arrow indicating a clockwise direction. To the right of this is an equals sign, followed by a zero.

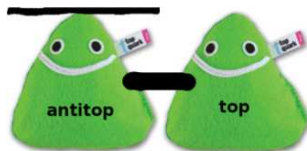
Gap Equation for NJL-model

Leading order N_c :

$$\begin{aligned} m_t &= -\frac{1}{2} G_t \langle \bar{t}t \rangle \\ &= 2G_t N_c m_t \frac{i}{(2\pi)^4} \int d^4l \frac{1}{l^2 - m_t^2}; \end{aligned}$$

Gap Equation in BCS theory

$$\Delta = \frac{V_0}{\hbar\beta V} \sum_{\mathbf{k}, n} -\frac{\hbar\Delta}{(\hbar\omega_n)^2 + (\varepsilon_{\mathbf{k}} - \mu)^2 + |\Delta|^2}.$$

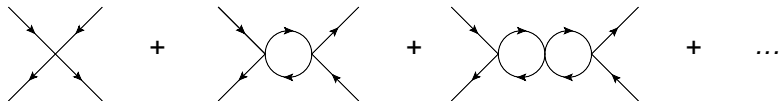


Gap equation:

$$m_t = m_t G_t \frac{N_c}{8\pi^2} \left[\Lambda^2 - m_t^2 \log \frac{\Lambda^2}{m_t^2} \right]$$

Valid for positive log: $G_t \geq \frac{8\pi^2}{N_c \Lambda^2} \equiv G_{\text{crit}}$.

Scalar bound state



Scalar channel

$$\begin{aligned}\Gamma_s(p^2) &= \int d^4x e^{ipx} \langle T \bar{t}t(0) \bar{t}t(x) \rangle_{\text{connected}} \\ &= -\frac{1}{2N_c} \left\{ (4m_t^2 - p^2) \frac{1}{16\pi^2} \right. \\ &\quad \left. \times \int_0^1 dx \log \left(\frac{\Lambda^2}{m_t^2 - x(1-x)p^2} \right) \right\}^{-1}.\end{aligned}$$

Scalar channel

$$\begin{aligned}\Gamma_s(p^2) &= \int d^4x e^{ipx} \langle T \bar{t}t(0) \bar{t}t(x) \rangle_{\text{connected}} \\ &= \frac{-1}{2N_c} \left\{ (4m_t^2 - p^2) \frac{1}{16\pi^2} \int_0^1 dx \log \left(\frac{\Lambda^2}{m_t^2 - x(1-x)p^2} \right) \right\}^{-1}\end{aligned}$$

Pole of Scalar Channel:

Scalar bound state with mass $2m_t = p$.

Integral with surface terms

$$\int \frac{d^4l}{(2\pi)^4} \frac{1}{(l^2 - m_t^2) ((p+l)^2 - m_t^2)}$$

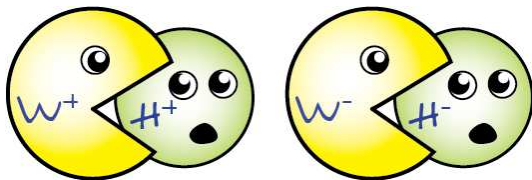
Massless Goldstone Bosons

From other channels

$$G_t(\bar{\psi}_L t_R)(\bar{t}_R \psi_L) = \frac{1}{4} G_t [(\bar{t}t)(\bar{t}t) + (\bar{t}i\gamma_5 t)(\bar{t}i\gamma_5 t) + (\bar{b}(1 + \gamma_5)t)(\bar{t}(1 - \gamma_5)b)],$$

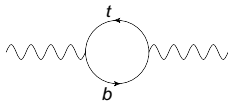
- Pseudoscalar Channel: $\langle T[\bar{t}i\gamma_5 t](x)[\bar{t}(-i)\gamma_5 t](0) \rangle$
- Charged Channels: $\langle T[\bar{t}(1 - \gamma_5)b](x)[\bar{b}(1 + \gamma_5)t](0) \rangle$, h.c.

All have poles at $p^2 = 0 \rightarrow$ Massless Goldstone Bosons.



Mass of the W boson

Renormalized W propagator: $\frac{-i(g_{\mu\nu} - p_\mu p_\nu / p^2)}{p^2} \left(\frac{1}{1 - \Pi_W(p^2)} \right)$



$$[\Pi_W^{\mu\nu}]_2 = \frac{ig}{\sqrt{2}} N_c \int \frac{d^4 l}{(2\pi)^4} (-1) \text{Tr} \left[\frac{i}{l + \not{p}} \gamma^\mu \frac{i}{l - m_t} \right] i\Gamma_c(p^2) \\ \times \frac{ig}{\sqrt{2}} N_c \int \frac{d^4 l}{(2\pi)^4} (-1) \text{Tr} \left[\frac{i}{l + \not{p}} \gamma^\nu \frac{i}{l - m_t} \right]$$

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ρ parameter in SM

$$\frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = \frac{M_W^2 \sqrt{g^2 + (g')^2}}{gM_Z^2} \equiv \rho$$

- SM: global $SO(4) = SU(2)_L \otimes SU(2)_R$ symmetry
- Broken by fermion masses to $SU(2) \equiv$ custodial symmetry
- Custodial symmetry ensures $\rho \simeq 1$

Higgs Matrix

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^0 & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}$$

$$\mathcal{L}_{Higgs} = \frac{1}{2} \text{Tr}[(D_\mu \Phi)^\dagger D^\mu \Phi] + \mu^2 \text{Tr}[\Phi^\dagger \Phi] - \lambda \text{Tr}[\Phi^\dagger \Phi \Phi^\dagger \Phi]$$

$$D_\mu \Phi = \partial_\mu \Phi + \frac{1}{2} ig W_\mu^a \tau_a \Phi - \frac{1}{2} ig' B_\mu \Phi \tau_3.$$

- Global $SO(4)$ Broken explicitly by $\frac{1}{2} ig' B_\mu \Phi \tau_3$ to $SU(2) \equiv$ custodial symmetry
- \rightarrow Higgs scalar is no special case of global $SU(2)$

Custodial symmetry holds

Find similar ρ -parameter



Figure: *Dance of the gauge bosons in vacuum* by Dr. Regina Valluzzi

W and Z masses: at $p^2 = M^2$

Pole of propagator: self-energy

$$\frac{-i(g_{\mu\nu} - p_\mu p_\nu / p^2)}{p^2} \left(\frac{1}{1 - \Pi_W(p^2)} \right)$$
$$\frac{1 - \Pi_W(p^2)}{(g')^2} = \frac{1}{(\bar{g}'(p^2))^2} - \frac{\bar{f}^2(p^2)}{p^2}.$$

ρ parameter

$$M_W^2 = p^2 = \bar{g}^2(M_W^2) \bar{f}^2(M_W^2)$$
$$M_Z^2 = f(M_Z^2) [g^2(M_Z^2) + (g')^2(M_Z^2)].$$

so that

$$\frac{M_W^2}{M_Z^2} \left(\frac{g^2(M_Z^2) + g'^2(M_Z^2)}{(\bar{g}')^2(M_W^2)} \right) = \frac{\bar{f}^2(M_W^2)}{f^2(M_Z^2)}.$$

S-, T-, and U-parameters

$$\alpha S = 4s_w^2 c_w^2 \left[\Pi'_{ZZ}(0) - \frac{c_w^2 - s_w^2}{s_w c_w} \Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0) \right]$$

$$\alpha T = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}$$

$$\alpha U = 4s_w^2 \left[\Pi'_{WW}(0) - c_w^2 \Pi'_{ZZ}(0) - 2s_w c_w \Pi'_{Z\gamma}(0) - s_w^2 \Pi'_{\gamma\gamma}(0) \right]$$

ρ -parameter in T

$$\rho = 1 + \delta\rho_{SM} + \alpha T.$$

Condensates can break electroweak symmetry:

- BCS theory: Cooper pairs give photon mass
- NJL model: Quark condensate gives gauge bosons mass
- Goldstone bosons and particle of mass $2m_t$ appear
- Custodial symmetry could still hold

Problems

- N_c limit
- UV-completion (above Λ) unknown
- Top too light for EWSB
- Top Higgs < 300 GeV excluded if top-pion ≥ 150 GeV.

Solutions

- Topcolor-assisted Technicolor (TC2)
- Top Seesaw: add χ (extended: ω)

Top Seesaw

- Enhanced gluon fusion $gg \rightarrow t\bar{t}$;
- $t - \chi$ mixing: contribution to T ;
- $b - \omega$ mixing: contributions to $T, Z \rightarrow \bar{b}b$;
- Existing Monte-Carlo with Technicolor features.

