Strong Electroweak Symmetry Breaking
Generating Masses Dynamically

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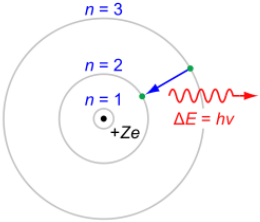
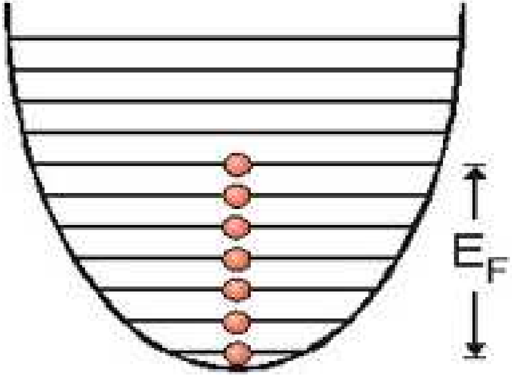
Overview

Old idea: Cooper pairs

Bardeen, Cooper, Schrieffer (1957)



Fermi Sea



Cooper pairs

+2 electrons

$\rightarrow$ Cooper: weak attraction gives bound state $< 2\varepsilon_F$

BCS Lagrangian and Condensate Approximation

Four-fermion Short-Ranged Interaction

$$\mathcal{L} = \sum_{\alpha=\uparrow,\downarrow} \phi^*_\alpha(\mathbf{x},\tau) \left\{ \hbar \frac{\partial}{\partial\tau} - \frac{\hbar^2\nabla^2}{2m_\alpha} - \mu_\alpha \right\} \phi_\alpha(\mathbf{x},\tau) + V_0 \phi^*_\uparrow(\mathbf{x},\tau)\phi^*_\downarrow(\mathbf{x},\tau)\phi_\downarrow(\mathbf{x},\tau)\phi_\uparrow(\mathbf{x},\tau)$$

Approximate

Pairing Field

$$\langle \Delta(\mathbf{x},\tau) \rangle = V_0 \langle \phi_\downarrow(\mathbf{x},\tau)\phi_\uparrow(\mathbf{x},\tau) \rangle \,.$$

Fluctuation Expansion

Approximate

Pairing Field Fluctuation

$$\Delta(\mathbf{x},\tau) = \Delta + \Delta'(\mathbf{x},\tau)$$

- Δ = minimum
- $\rightarrow$ linear terms vanish.
- $\rightarrow \mathcal{L}_{\text{eff}}(\Delta'^{*}, \Delta', \Delta^{*}, \Delta)$

The Gap

Propagator pole

$$\mathbf{G}_\Delta(\mathbf{k}, i\omega_n) = \frac{-\hbar}{-(\hbar\omega_n)^2 - [(\varepsilon_\mathbf{k} - \mu)^2 + |\Delta|^2]} \begin{bmatrix} -i\hbar\omega_n - (\varepsilon_\mathbf{k} - \mu) & \Delta \\ \Delta^* & -i\hbar\omega_n + \varepsilon_\mathbf{k} - \mu \end{bmatrix}.$$

$$\hbar\omega = \sqrt{(\varepsilon_\mathbf{k} - \mu)^2 + |\Delta|^2} \equiv \hbar\omega_\mathbf{k}$$

.

Minimum of pole (when $\varepsilon_k^2 = \frac{\hbar^2 k^2}{2m} = \mu$) = $|\Delta|$

$\rightarrow$ Break Cooper pair with 2Δ = gap.

Linear terms vanish:

Gap Equation

In Fourier Space:

$$\Delta = \frac{V_0}{\hbar\beta V} \sum_{\mathbf{k},n} - \frac{\hbar\Delta}{(\hbar\omega_n)^2 + (\varepsilon_{\mathbf{k}} - \mu)^2 + |\Delta|^2}.$$

Overview

Nambu–Jona-Lasinio Model

Four-fermion interaction

$$
\begin{aligned}
\mathcal{L} &= i\bar{\psi}\not{D}\psi + G_t(\bar{\psi}_L t_R)(\bar{t}_R \psi_L). \\
\psi &= \begin{bmatrix} t \\ b \end{bmatrix} \\
\not{D} &= D^\mu \gamma_\mu = \left[\partial^\mu + i\frac{g_s}{2}G_a^\mu L_a + i\frac{g}{2}W_b^\mu \sigma_b + i\frac{g'}{2}B^\mu Y\right]\gamma_\mu.
\end{aligned}
$$

Rewrite Lagrangian:

$$
\begin{aligned}
\mathcal{L}'_0 &= \bar{t}i\gamma^\mu\partial_\mu t + \bar{b}i\gamma^\mu\partial_\mu b - m_t\bar{t}t \\
\mathcal{L}'_I &= m_t\bar{t}t + G_t(\bar{\psi}_L t_R)(\bar{t}_R\psi_L).
\end{aligned}
$$

Dynamical Mass Generation

- Start without bare mass: $m_0 = 0$;
- Insert physical mass in propagator: $\frac{1}{\not{p} - m_P - \Sigma} \rightarrow$ demand $\Sigma = 0$.

Interactions

$$\mathcal{L}'_I = m_t \bar{t} t + G_t (\bar{\psi}_L t_R)(\bar{t}_R \psi_L).$$

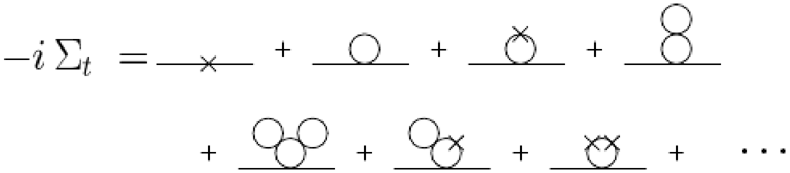


Figure: From Cristian Valenzuela Roubillard (thesis, 2005)

Gap Equation

Gap Equation for NJL-model

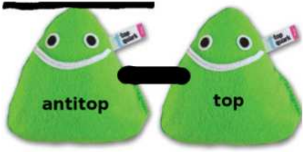
Leading order N_c:

$$\begin{aligned} m_t &= -\frac{1}{2} G_t \left\langle \bar{t} t \right\rangle \\ &= 2 G_t N_c m_t \frac{i}{(2\pi)^4} \int d^4 l \frac{1}{l^2 - m_t^2}; \end{aligned}$$

Gap Equation in BCS theory

$$\Delta = \frac{V_0}{\hbar \beta V} \sum_{\mathbf{k},n} - \frac{\hbar \Delta}{(\hbar \omega_n)^2 + (\varepsilon_{\mathbf{k}} - \mu)^2 + |\Delta|^2}.$$

Nonzero top mass

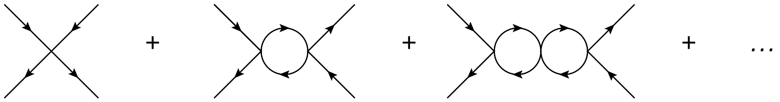


Gap equation:

$$m_t = m_t G_t \frac{N_c}{8\pi^2}\left[\Lambda^2 - m_t^2 \log\frac{\Lambda^2}{m_t^2}\right]$$

Valid for positive log: $G_t \geq \frac{8\pi^2}{N_c\Lambda^2} \equiv G_{\text{crit}}$.

Scalar bound state



Scalar channel

$$\Gamma_S(p^2) = \int d^4x\, e^{ipx} \langle 0|T[\bar{t}t](x)[\bar{t}t](0)|0\rangle$$

$$= -\frac{1}{2N_c}\left\{(4m_t^2 - p^2)\frac{1}{16\pi^2} \times \int_0^1 dx \log\left(\frac{\Lambda^2}{m_t^2 - x(1-x)p^2}\right)\right\}^{-1}.$$

Scalar bound state

Scalar channel

$$\Gamma_s(p^2) = \int d^4x e^{ipx} \langle 0|T[\bar{t}t](x)[\bar{t}t](0)|0\rangle$$
$$= \frac{-1}{2N_c}\left\{(4m_t^2 - p^2)\frac{1}{16\pi^2}\int_0^1 dx \log\left(\frac{\Lambda^2}{m_t^2 - x(1-x)p^2}\right)\right\}^{-1}$$

Pole of Scalar Channel:

Scalar bound state with mass $2m_t = p$.

Integral with surface terms

$$\int \frac{d^4l}{(2\pi)^4}\frac{1}{(p+l)^2 - m_t^2} = \int \frac{d^4l}{(2\pi)^4}\frac{1}{l^2 - m_t^2} - \frac{ip^2}{32\pi^2},$$

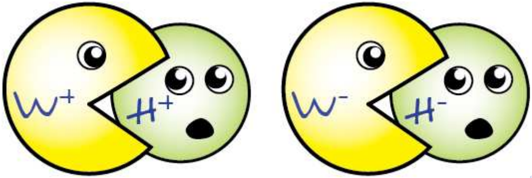
Massless Goldstone Bosons

From other channels

$$G_t(\bar{\psi}_L t_R)(\bar{t}_R \psi_L) = \frac{1}{4}G_t\left[(\bar{t}t)(\bar{t}t) + (\bar{t}i\gamma_5 t)(\bar{t}i\gamma_5 t) + (\bar{b}(1+\gamma_5)t)(\bar{t}(1-\gamma_5)b))\right],$$

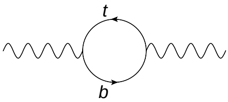
- Pseudoscalar Channel: $\langle T[\bar{t}i\gamma_5 t](x)[\bar{t}(-i)\gamma_5 t](0)\rangle$
- Charged Channels: $\langle T[\bar{t}(1-\gamma_5)b](x)[\bar{b}(1+\gamma_5)t](0)\rangle$, h.c.

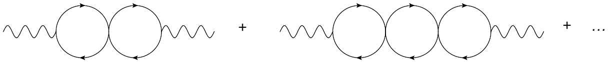
All have poles at $p^2 = 0 \rightarrow$ Massless Goldstone Bosons.



Mass of the W boson

Renormalized W propagator: $\frac{-i(g_{\mu\nu}-p_\mu p_\nu/p^2)}{p^2}\left(\frac{1}{1-\Pi_W(p^2)}\right)$





$$
\begin{aligned}
[\Pi_W^{\mu\nu}]_2 \quad &= \quad \frac{ig}{\sqrt{2}} N_c \int \frac{d^4 l}{(2\pi)^4} (-1)\mathrm{Tr}\left[\frac{i}{\not{l}+\not{p}}\gamma^\mu \frac{i}{\not{l}-m_t}\right] i\Gamma_c(p^2) \\
&\times \frac{ig}{\sqrt{2}} N_c \int \frac{d^4 l}{(2\pi)^4} (-1)\mathrm{Tr}\left[\frac{i}{\not{l}+\not{p}}\gamma^\nu \frac{i}{\not{l}-m_t}\right]
\end{aligned}
$$

Overview

Custodial Symmetry in SM

ρ parameter in SM

$$\frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \frac{M_W^2\sqrt{g^2+(g')^2}}{gM_Z^2} \equiv \rho$$

- SM: global $SO(4) = SU(2)_L \otimes SU(2)_R$ symmetry
- Broken by fermion masses to $SU(2) \equiv$ custodial symmetry
- Custodial symmetry ensures $\rho \simeq 1$

Custodial Symmetry: SM in different form

Higgs Matrix

$$
\begin{aligned}
\Phi &= \begin{pmatrix} \phi^{0*} & \phi^{+} \\ -\phi^{+*} & \phi^{0} \end{pmatrix} = \begin{pmatrix} \phi^{0} & \phi^{+} \\ -\phi^{-} & \phi^{0} \end{pmatrix} \\
\mathcal{L}_{Higgs} &= \frac{1}{2}\mathrm{Tr}[(D_\mu \Phi)^\dagger D^\mu \Phi] + \mu^2 \mathrm{Tr}[\Phi^\dagger \Phi] - \lambda \mathrm{Tr}[\Phi^\dagger \Phi \Phi^\dagger \Phi] \\
D_\mu \Phi &= \partial_\mu \Phi + \frac{1}{2} i g W_\mu^a \tau_a \Phi - \frac{1}{2} i g' B_\mu \Phi \tau_3 .
\end{aligned}
$$

- Global $SO(4)$ Broken explicitly by $\frac{1}{2} i g' B_\mu \Phi \tau_3$ to $SU(2) \equiv$ custodial symmetry
- $\rightarrow$ Higgs scalar is no special case of global $SU(2)$

W and Z masses in NJL model

Custodial symmetry holds

Find similar ρ-parameter

Figure: *Dance of the gauge bosons in vacuum* by Dr. Regina Valluzzi

W and Z masses: at $p^2 = M^2$

Pole of propagator: self-energy

$$\frac{-i(g_{\mu\nu} - p_\mu p_\nu / p^2)}{p^2} \left(\frac{1}{1 - \Pi_W(p^2)} \right)$$

$$\frac{1 - \Pi_W(p^2)}{(g')^2} = \frac{1}{(\bar{g}'(p^2))^2} - \frac{\bar{f}^2(p^2)}{p^2}.$$

ρ parameter

$$M_W^2 = p^2 = \bar{g}^2(M_W^2)\bar{f}^2(M_W^2)$$

$$M_Z^2 = f(M_Z^2)[g^2(M_Z^2 + (g')^2(M_Z^2)].$$

so that

$$\frac{M_W^2}{M_Z^2} \left(\frac{g^2(M_Z^2) + g(')^2(M_Z^2)}{(\bar{g}')^2(M_W^2)} \right) = \frac{\bar{f}^2(M_W^2)}{f^2(M_Z^2)}.$$

Peskin-Takeuchi parameters

S-, T-, and U-parameters

$$\begin{aligned}
\alpha S &= 4s_W^2 c_W^2 \left[\Pi'_{ZZ}(0) - \frac{c_W^2 - s_W^2}{s_W c_W}\Pi'_{Z\gamma}(0) - \Pi'_{\gamma\gamma}(0)\right] \\
\alpha T &= \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} \\
\alpha U &= 4s_W^2 \left[\Pi'_{WW}(0) - c_W^2\Pi'_{ZZ}(0) - 2s_W c_W \Pi'_{Z\gamma}(0) - s_W^2 \Pi'_{\gamma\gamma}(0)\right]
\end{aligned}$$

ρ-parameter in T

$$\rho = 1 + \delta\rho_{SM} + \alpha T\,.$$

Overview

Topcolor

The Gauge Group

$$SU(3)_{1/2} \otimes SU(3)_3 \otimes U(1)_{1/2} \otimes U(1)_3 \otimes SU(2)_L$$
$$\overset{\langle H_t \rangle = f_{\pi_t}}{\longrightarrow} SU(3)_{QCD} \otimes U(1)_{EM}$$

Problems

- Top too light for EWSB
- N_c limit
- UV-completion (above Λ) unknown
- Top Higgs $\langle \bar{t}t \rangle \equiv H_t < 300$ GeV excluded if top-pion ≥ 150 GeV.

$\neq$ 

Example: Minimal model (Susskind, Weinberg)

$$SU(N_T) \times SU(3) \times SU(2)_L \times U(1)_Y$$

Problems

- Multiple generations (Extended Technicolor)
- Electroweak Precision Measurements (Walking Technicolor)
- Heavy quark masses (Walking Technicolor)

Solutions

Viable theories

- Topcolor-assisted Technicolor (TC2)
- Top Seesaw: add χ (extended: ω)

Overview

Topcolor-assisted technicolor (TC2)

TC2

$$SU(3)_{1/2} \otimes SU(3)_3 \otimes U(1)_{1/2} \otimes U(1)_3 \stackrel{\langle\Phi\rangle}{\longrightarrow} SU(3)_{QCD} \otimes U(1)_Y$$

$$SU(2)_L \otimes U(1)_Y \stackrel{\langle H\rangle = f_T, \langle H_t\rangle = f_{\pi_t}}{\longrightarrow} U(1)_{EM}$$

Combined VEV

$$v_{\mathrm{ew}}^2 = f_T^2 + f_\pi^2$$

TC2 Effective theory: Auxiliary Fields

Auxiliary Fields

$$\mathcal{L} = i\bar{\psi}\not{D}\psi + ch\bar{\psi}\psi - m^2h^2 \rightarrow i\bar{\psi}\not{D}\psi + c^2(\bar{\psi}\psi)^2$$

For NJL model:

$$\bar{\psi}i\not{D}\psi + G_t\bar{\psi}_L t_R \bar{t}_R \psi_L \rightarrow \bar{\psi}i\not{D}\psi - m_{t0}^2 H_t^\dagger H_t + g_{t0}(\bar{\psi}_L t_R H_t + \text{h.c.})$$

TC2 Effective theory: Linear Sigma Model

Linear Sigma Model

$$
\begin{aligned}
H_t &= \exp\left(\frac{i\pi_t^a\tau_a}{\sqrt{2}f_{\pi_t}}\right)\begin{pmatrix} f_{\pi_t} + \frac{1}{\sqrt{2}}h_t \\ 0 \end{pmatrix} \\
\mathcal{L}_\Sigma &= g_{t0}(\bar{\psi}_L t_R H_t + \text{h.c.}) \approx \frac{\lambda_t}{\sqrt{2}}(f_{\pi_t}\bar{t}_L t_R + \bar{t}_L i\pi_t^0 t_R + \bar{t}_L h_t t_R) + h.c. + \dots
\end{aligned}
$$

TC2 Effective Theory: CKM-constrained mixing

$$M_U = f_T \begin{pmatrix} Y_{uU} & Y_{uC} & \sim 0 \\ Y_{cU} & Y_{cC} & \sim 0 \\ c_{Tu}\varepsilon & c_{Tc}\varepsilon & +\frac{m_t}{f_T} \end{pmatrix}$$

Quark mixing: Diagonalize Mass Matrix

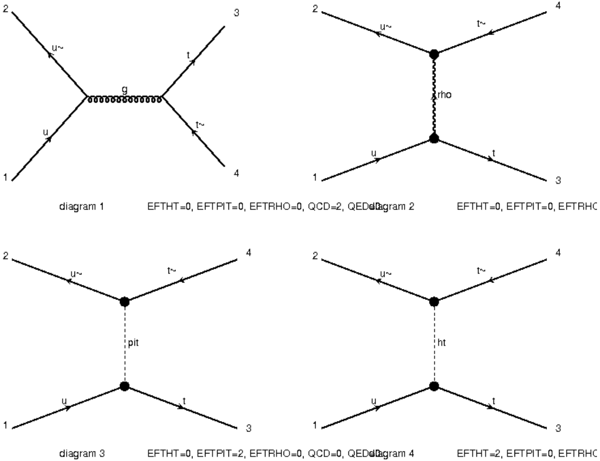
$$t_R \to V_{11}^{Ru} t_R + V_{21}^{Ru} c_R + V_{31}^{Ru} u_R.$$

Effective Lagrangian

$$\mathcal{L}_{A^t_{FB}} = g_{tu\pi_t} i\pi_t^0 \bar{t}_L u_R + g_{tuh_t} h_t \bar{t}_L u_R + g_{tu\rho} \rho_\mu \bar{t}_R \gamma^\mu u_R + h.c.$$

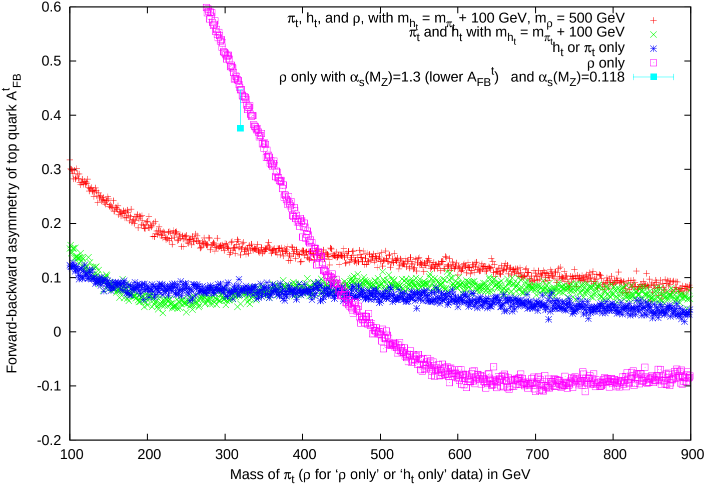
Forward-Backard Asymmetry of the Top

$$\mathcal{L}_{A^t_{FB}} = g_{t0}(\bar{\psi}_L t_R H_t + \text{h.c.}) \rightarrow g_{\pi_t} i \pi_t^0 \bar{t}_L u_R + g_{tuh_t} h_t \bar{t}_L u_R + g_\rho \rho_\mu \bar{t}_R \gamma^\mu u_R + h.c$$

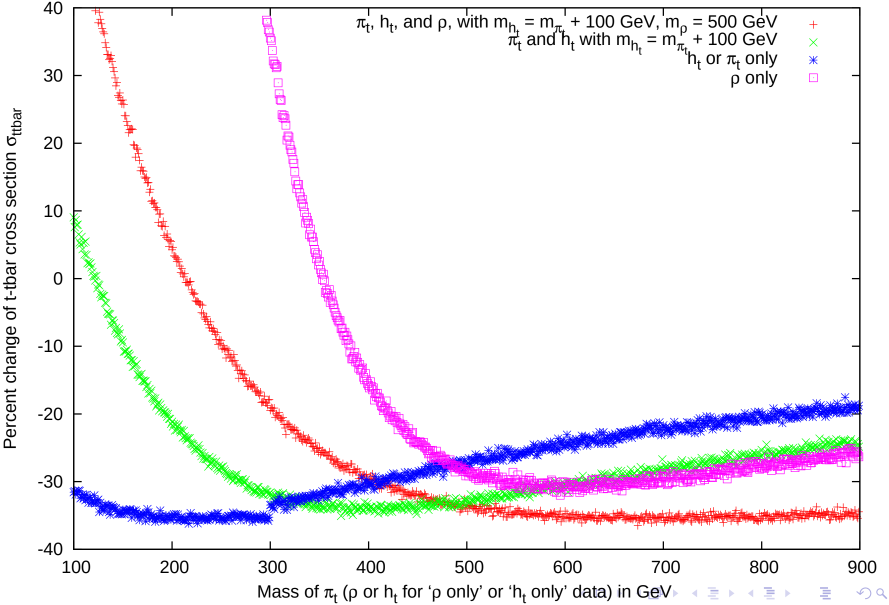


A^t_{FB} -Forward-Backard Asymmetry of the Top

Tevatron: 0.030 ± 0.007 for $\bar{t}t +$ bg; measured $A^t_{FB} = 0.210 \pm 0.049$



$\Delta\sigma_{t\bar{t}}$ - Deviation of cross section



Conclusion

Condensates can break electroweak symmetry:

- BCS theory: Cooper pairs give photon mass
- NJL model: Quark condensate gives gauge bosons mass
- Goldstone bosons and particle of mass $2m_t$ appear
- Custodial symmetry could still hold
- TC2 still viable
- A_{FB}^{t} could be explained by TC2

Discussion

- Custodial symmetry in NJL model by defining parameters
- A^t_{FB} could be explained by other models
- ρ Lagrangian added by hand
- What is Φ in TC2?
- ...