Strong Electroweak Symmetry Breaking
Generating Masses Dynamically

Supervisor: Prof. Eric Laenen

Jory Sonneveld

Nikhef / Utrecht University

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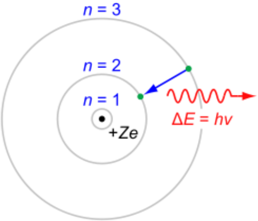
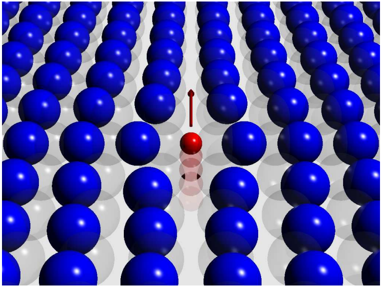
Layout

Old idea: Cooper pairs

Bardeen, Cooper, Schrieffer (1957)



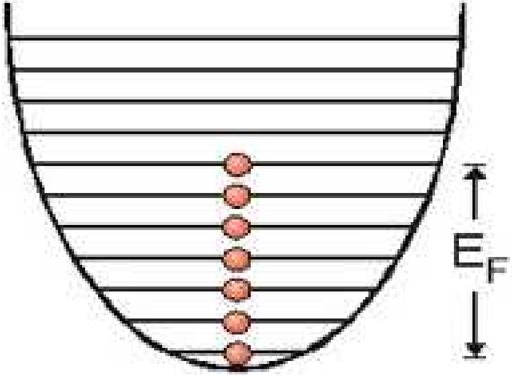
Electrons in a Lattice



Phonon: Scattering mediators

photon:light wave = phonon:sound wave

Fermi Sea



Cooper pairs

+2 electrons

$\rightarrow$ Cooper: weak attraction gives bound state $< 2\varepsilon_F$

BCS Lagrangian and Condensate Approximation

Four-fermion Short-Ranged Interaction

$$\mathcal{L} = \sum_{\alpha=\uparrow,\downarrow} \phi^*_\alpha(\mathbf{x},\tau)\left\{\hbar\frac{\partial}{\partial\tau} - \frac{\hbar^2\nabla^2}{2m_\alpha} - \mu_\alpha\right\}\phi_\alpha(\mathbf{x},\tau)$$
$$+V_0\phi^*_\uparrow(\mathbf{x},\tau)\phi^*_\downarrow(\mathbf{x},\tau)\phi_\downarrow(\mathbf{x},\tau)\phi_\uparrow(\mathbf{x},\tau)$$

Approximate

Pairing Field

$$\langle\Delta(\mathbf{x},\tau)\rangle = V_0\langle\phi_\downarrow(\mathbf{x},\tau)\phi_\uparrow(\mathbf{x},\tau)\rangle\,.$$

Fluctuation Expansion

Approximate

Pairing Field Fluctuation

$$\Delta(\mathbf{x}, \tau) = \Delta + \Delta'(\mathbf{x}, \tau)$$

- Δ = minimum
- $\rightarrow$ linear terms vanish.
- $\rightarrow \mathcal{L}_{\text{eff}}(\Delta'^{*}, \Delta', \Delta^{*}, \Delta)$

The Gap

Propagator pole

$$\mathbf{G}_\Delta(\mathbf{k}, i\omega_n) = \frac{-\hbar}{-(\hbar\omega_n)^2 - [(\varepsilon_\mathbf{k} - \mu)^2 + |\Delta|^2]} \begin{bmatrix} -i\hbar\omega_n - (\varepsilon_\mathbf{k} - \mu) & \Delta \\ \Delta^* & -i\hbar\omega_n + \varepsilon_\mathbf{k} - \mu \end{bmatrix}.$$

$$\hbar\omega = \sqrt{(\varepsilon_\mathbf{k} - \mu)^2 + |\Delta|^2} \equiv \hbar\omega_\mathbf{k}$$

.

Minimum of pole (when $\varepsilon_k^2 = \frac{\hbar^2 k^2}{2m} = \mu$) = $|\Delta|$

$\rightarrow$ Break Cooper pair with 2Δ = gap.

Gap Equation

Linear terms vanish:

Gap Equation

In Fourier Space:

$$\Delta = \frac{V_0}{\hbar\beta V}\sum_{\mathbf{k},n} - \frac{\hbar\Delta}{(\hbar\omega_n)^2 + (\varepsilon_{\mathbf{k}} - \mu)^2 + |\Delta|^2}.$$

Layout

Four-fermion interaction

$$\mathcal{L} = \bar{\psi} \not{D} \psi + G_t (\bar{\psi}_L t_R)(\bar{t}_R \psi_L).$$

$$\psi = \begin{bmatrix} t \\ b \end{bmatrix}$$

$$\not{D} = D^\mu \gamma_\mu = \left[\partial^\mu + i \frac{g_s}{2} G^\mu_a L_a + i \frac{g}{2} W^\mu_b \sigma_b + i \frac{g'}{2} B^\mu Y \right] \gamma_\mu.$$

Dynamical Mass Generation

- Start without bare mass: $m_0 = 0$;
- Insert physical mass in propagator: $\frac{1}{\not{p} - m_P - \Sigma} \longrightarrow$ demand $\Sigma = 0$.

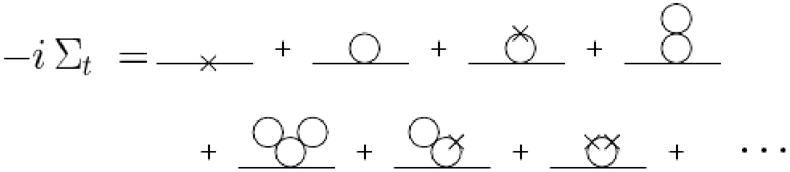


Figure: From Cristian Valenzuela Roubillard (thesis, 2005)

Gap Equation

Gap Equation for NJL-model

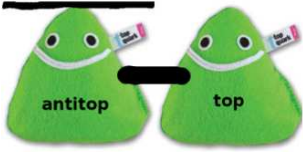
Leading order N_c:

$$
\begin{aligned}
m_t &= -\frac{1}{2} G_t \left\langle \bar{t} t \right\rangle \\
&= 2 G_t N_c m_t \frac{i}{(2\pi)^4} \int d^4 l \frac{1}{l^2 - m_t^2};
\end{aligned}
$$

Gap Equation in BCS theory

$$
\Delta = \frac{V_0}{\hbar \beta V} \sum_{\mathbf{k},n} - \frac{\hbar \Delta}{(\hbar \omega_n)^2 + (\varepsilon_{\mathbf{k}} - \mu)^2 + |\Delta|^2}.
$$

Coupling G_t

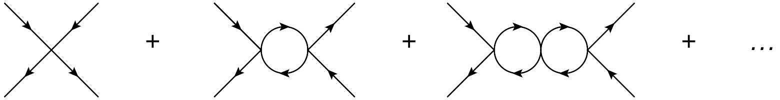


Gap equation:

$$m_t = m_t G_t \frac{N_c}{8\pi^2}\left[\Lambda^2 - m_t^2 \log\frac{\Lambda^2}{m_t^2}\right]$$

Valid for positive log: $G_t \geq \frac{8\pi^2}{N_c\Lambda^2} \equiv G_{\text{crit}}$.

Scalar bound state



Scalar channel

$$
\begin{aligned}
\Gamma_s(p^2) &= \int d^4x\, e^{ipx} \left\langle T\bar{t}t(0)\bar{t}t(x)\right\rangle_{\text{connected}} \\
&= -\frac{1}{2N_c}\left\{(4m_t^2 - p^2)\frac{1}{16\pi^2} \right. \\
&\qquad \left. \times \int_0^1 dx \log\left(\frac{\Lambda^2}{m_t^2 - x(1-x)p^2}\right)\right\}^{-1} .
\end{aligned}
$$

Scalar bound state

Scalar channel

$$\Gamma_s(p^2) = \int d^4x e^{ipx} \langle T\bar{t}t(0)\bar{t}t(x)\rangle_{\text{connected}}$$

$$= \frac{-1}{2N_c}\left\{(4m_t^2 - p^2)\frac{1}{16\pi^2}\int_0^1 dx \log\left(\frac{\Lambda^2}{m_t^2 - x(1-x)p^2}\right)\right\}^{-1}$$

Pole of Scalar Channel:

Scalar bound state with mass $2m_t = p$.

Integral with surface terms

$$\int \frac{d^4l}{(2\pi)^4}\frac{1}{(l^2 - m_t^2)\left((p+l)^2 - m_t^2\right)}$$

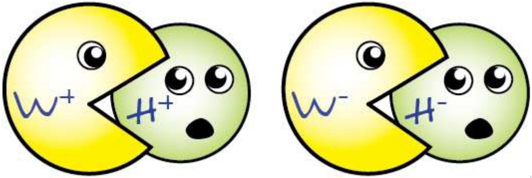
Massless Goldstone Bosons

From other channels

$$G_t(\bar{\psi}_L t_R)(\bar{t}_R \psi_L) = \frac{1}{4} G_t \left[(\bar{t}t)(\bar{t}t) + (\bar{t}i\gamma_5 t)(\bar{t}i\gamma_5 t) + (\bar{b}(1+\gamma_5)t)(\bar{t}(1-\gamma_5)b)) \right],$$

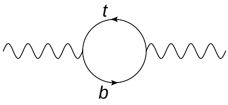
- Pseudoscalar Channel: $\langle T[\bar{t}i\gamma_5 t](x)[\bar{t}(-i)\gamma_5 t](0)\rangle$
- Charged Channels: $\langle T[\bar{t}(1-\gamma_5)b](x)[\bar{b}(1+\gamma_5)t](0)\rangle$, h.c.

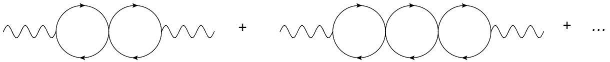
Both have poles at $p^2 = 0 \rightarrow$ Massless Goldstone Bosons.



Mass of the W boson

Renormalized W propagator: $\frac{-i(g_{\mu\nu}-p_\mu p_\nu/p^2)}{p^2}\left(\frac{1}{1-\Pi_W(p^2)}\right)$





$$[\Pi_W^{\mu\nu}]_2 = \frac{ig}{\sqrt{2}} N_c \int \frac{d^4 l}{(2\pi)^4}(-1)\mathrm{Tr}\left[\frac{i}{\not{l}+\not{p}}\gamma^\mu \frac{i}{\not{l}-m_t}\right] i\Gamma_c(p^2)$$
$$\times \frac{ig}{\sqrt{2}} N_c \int \frac{d^4 l}{(2\pi)^4}(-1)\mathrm{Tr}\left[\frac{i}{\not{l}+\not{p}}\gamma^\nu \frac{i}{\not{l}-m_t}\right]$$

Layout

Custodial Symmetry in SM

ρ parameter in SM

$$\frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \frac{M_W^2\sqrt{g^2+(g')^2}}{gM_Z^2} \equiv \rho$$

- SM: global $SO(4) = SU(2)_L \otimes SU(2)_R$ symmetry
- Broken by fermion masses to $SU(2) \equiv$ custodial symmetry
- Custodial symmetry ensures $\rho \simeq 1$

Custodial Symmetry: SM in different form

Higgs Matrix

$$\begin{aligned}\Phi &= \begin{pmatrix} \phi^{0*} & \phi^{+} \\ -\phi^{+*} & \phi^{0} \end{pmatrix} = \begin{pmatrix} \phi^{0} & \phi^{+} \\ -\phi^{-} & \phi^{0} \end{pmatrix} \\ \mathcal{L}_{Higgs} &= \frac{1}{2}\mathrm{Tr}[(D_\mu\Phi)^\dagger D^\mu\Phi] + \mu^2\mathrm{Tr}[\Phi^\dagger\Phi] - \lambda\mathrm{Tr}[\Phi^\dagger\Phi\Phi^\dagger\Phi] \\ D_\mu\Phi &= \partial_\mu\Phi + \frac{1}{2}igW_\mu^a\tau_a\Phi - \frac{1}{2}ig'B_\mu\Phi\tau_3.\end{aligned}$$

- Global $SO(4)$ Broken by $\frac{1}{2}ig'B_\mu\Phi\tau_3$ to $SU(2) \equiv$ custodial symmetry
- $\rightarrow$ Higgs scalar is no special case of global $SU(2)$

W and Z masses in NJL model

Custodial symmetry holds

Find similar ρ-parameter

Figure: *Dance of the gauge bosons in vacuum* by Dr. Regina Valluzzi

W and Z masses: at $p^2 = M^2$

Pole of propagator: self-energy

$$\frac{-i(g_{\mu\nu} - p_\mu p_\nu/p^2)}{p^2}\left(\frac{1}{1-\Pi_W(p^2)}\right)$$

$$\frac{1-\Pi_W(p^2)}{(g')^2} = \frac{1}{(\bar{g}'(p^2))^2} - \frac{\bar{f}^2(p^2)}{p^2}.$$

ρ parameter

$$M_W^2 = p^2 = \bar{g}^2(M_W^2)\bar{f}^2(M_W^2)$$

$$M_Z^2 = f(M_Z^2)[g^2(M_Z^2 + (g')^2(M_Z^2)].$$

so that

$$\frac{M_W^2}{M_Z^2}\left(\frac{g^2(M_Z^2) + g(')^2(M_Z^2)}{(\bar{g}')^2(M_W^2)}\right) = \frac{\bar{f}^2(M_W^2)}{f^2(M_Z^2)}.$$

Conclusion

Condensates can break electroweak symmetry:

- BCS theory: Cooper pairs give photon mass
- NJL model: Quark condensate gives gauge bosons mass
- Goldstone bosons and particle of mass $2m_t$ appear
- Custodial symmetry could still hold

Further work

Next: phenomenology in LHC

- Other theories using strong top dynamics: topcolor-assisted technicolor, top-seesaw models
- Top-Higgs < 300 GeV excluded if top-pion ≥ 150 GeV.
- Gluon fusion $gg \rightarrow t\bar{t}$
- Existing Monte-Carlo program to test new vertices.