

# Neutrino physics at the LHC

by

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Ich versichere hiermit, dass ich die Arbeit selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt sowie Zitate kenntlich gemacht habe.

Aachen, den 26.08.2013

### Zusammenfassung

Die Ergebnisse neuerer Forschung haben gezeigt, dass das Neutrino kein masseloses Teilchen ist. Dies macht es notwendig, die Natur des Neutrinos zu untersuchen. Ziel dieser Arbeit ist es, anzugeben wie viele neutrinolose doppelte Betazerfall Ereignisse am LHC bei Proton-Proton Kollisionen mit  $\sqrt{s} = 14 \text{ TeV}$  sichtbar sind, wenn man einen Type III Seesaw Mechanismus annimmt. Das Auftreten eines solchen, die Erhaltung der Leptonenzahl verletzenden Prozesses, ist ein Anzeichen für die mögliche Majorana Natur des Neutrinos.

Um dies zu zeigen wird der Wirkungsquerschnitt eines Kollisionsprozesses berechnet. Ergebnisse der neuesten Experimente werden erläutert und dahingehend untersucht, mit welchen Werten im Rahmen der Annahmen die größtmögliche Menge an Ereignissen erzielt wird. Die Berechnung des Wirkungsquerschnittes für den neutrinolosen doppelten Betazerfall erfolgt mit MadGraph. Wir haben ermittelt, dass es in der  $23 \text{ fb}^{-1}$  großen Sammlung integrierter Luminosität des LHC von 2012 genau ein solches Ereignis gibt.

### Abstract

Current results of research have shown that the neutrino is not a massless particle. This raises the need to investigate the neutrino's nature. This thesis' aim is to provide a number of possible neutrinoless double beta decay events observable in proton-proton collisions at the LHC running at  $\sqrt{s} = 14 \text{ TeV}$  assuming a Type III seesaw mechanism. The existence of this lepton number conservation violating signal would yield evidence that the neutrino is a Majorana particle.

Therefore it is shown how to calculate the cross section of a collision by hand. Data from recent experiments is stated and discussed in order to obtain values which leads to the highest possible number of events. The neutrinoless double beta decay calculation is conducted using MadGraph. We conclude that there is one event observable in the LHC's collection of  $23 \text{ fb}^{-1}$  integrated luminosity from 2012.

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## 1 Introduction

Neutrinoless double beta decay ( $0\nu\beta\beta$ ) is a very rare process, if even possible. The purpose of this thesis is to calculate how rare. It will be stated how exactly  $0\nu\beta\beta$  is possible and which assumptions are necessary for it to happen. This thesis aims to give a maximum number of events with  $0\nu\beta\beta$  that could be visible at the LHC with the center-of-mass energy  $\sqrt{s} = 14$  TeV.

First, the actual insights on the nature of the neutrino will be discussed in this chapter. Next, the beta decay will be discussed in more detail.  $0\nu\beta\beta$  is not possible in the Standard model (SM). This makes it necessary to extend the SM. A model for the extension will be chosen and explained in the second chapter. In the third chapter, limits on the double beta decay with neutrinos ( $2\nu\beta\beta$ ) and  $0\nu\beta\beta$  half-lives from current experiments will be discussed. Values used in the calculation will be listed in the same section. The fourth chapter gives an overview on how to use Feynman rules, followed by the calculation of an example cross section. This example will be the collision of a  $u$  and a  $\bar{d}$  quark to obtain a  $W$  boson. In the last chapter the  $0\nu\beta\beta$ -Feynman diagram is rotated and viewed as a pp-collider LHC process. Here, the event count that we want to obtain will be calculated.

### 1.1 History of neutrinos

In the last decades, several experiments on the fluxes of atmospheric, solar and reactor neutrinos were conducted and provided evidence on the existence of neutrino oscillations. The phenomenon of neutrino oscillation was predicted in 1967 by Bruno Pontecorvo [1]. It states that a neutrino created with a certain lepton flavour can later be measured with a different one. This is only possible if the mass and the flavour eigenstates are not the same and non-zero. In quantum mechanics an eigenstate denotes one of the characteristic functions of an observable, here flavour and mass. Hence, it implies that neutrinos have a non-vanishing mass.

However, according to the Standard Model (SM) of particle physics - introduced

in the 1960s by Glashow, Weinberg and Salam [2] - neutrinos are supposed to be particles without mass. This is one of the shortcomings of the known model which makes extensions necessary and thus increases the interest in physics beyond the SM.

One idea for dealing with the massive neutrino was suggested by Ettore Majorana in 1937 [3]. As a so-called Majorana particle, neutrinos would be their own antiparticle. This leads to the existence of processes with violation of lepton number conservation which would allow phenomena such as the neutrinoless double beta decay. The nature of the Majorana particle and how it differs from the kind of particle used to describe neutrinos in the past is explained at the end of this chapter.

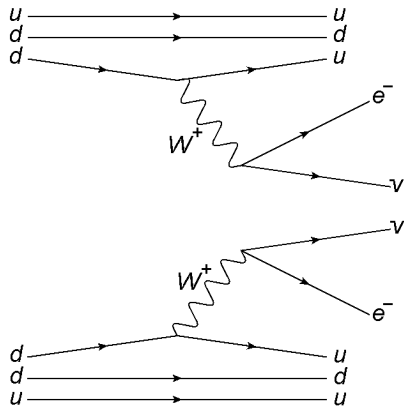
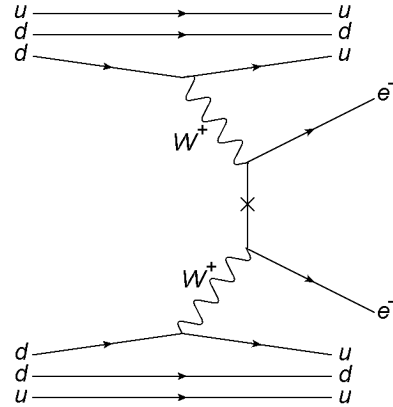
## 1.2 Beta decay modes

Much more common than  $0\nu\beta\beta$  is the simple beta decay. Beta minus ( $\beta^-$ ) decay is a radioactive process where a neutron in an atomic nucleus decays into a proton and emits an electron and an antineutrino

$$(A, Z) \rightarrow (A, Z + 1) + e^- + \bar{\nu}_e.$$

Here  $A$  is the mass number and  $Z$  is the atomic number of the decaying nucleus. Since two particles are emitted, the energy of the mass excess is allocated randomly on the electron and the antineutrino. This leads to a continuous spectrum for measurements of the electron energy. The apparent loss of energy violates the law of conservation and provided the first evidence for the existence of neutrinos.

An even-even nucleus consists of an even number of protons and an even number of neutrons. For some of these even-even nuclei where the single beta decay is energetically forbidden or at least highly suppressed, observation of double beta decay is possible. Single beta decay has to be suppressed in order to make the rare  $2\nu\beta\beta$  - depicted in Figure 1 - observable.  $2\nu\beta\beta$  is in essence just two single

Figure 1: Feynman diagram of  $2\nu\beta\beta$ Figure 2: Feynman diagram of  $0\nu\beta\beta$ . The  $\times$  marks the neutrino annihilation

beta decays in the same nucleus at the same time:

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e.$$

Of those 35 isotopes known to be capable of double beta decay, only in ten the decay has been observed experimentally [4, pp.631]. Depending on the isotope involved, the  $2\nu\beta\beta$  half-lives  $T_{2\nu\beta\beta}^{1/2}$  vary roughly between  $10^{19}$  and  $10^{21}$  years. These will be specified in section 4.1.

As mentioned before, neutrinos are now known to have a non-vanishing mass. From this arises the need for new models of physics beyond the SM. Several of the new models predict that lepton number will not be conserved anymore. If lepton number conservation could indeed be broken, then double beta decay without the two neutrinos - depicted in Figure 2 - would be possible:

$$(A, Z) \rightarrow (A, Z + 2) + e^- + e^-.$$

The disappearance could be explained by e.g. supersymmetric R-parity violation [5, p.105] or through adding extra dimensions. Assuming there are no new parti-



cles or dimensions, the neutrinos' disappearance could be explained if the neutrino is a Majorana particle.

### 1.3 Majorana, Dirac and Weyl

Besides Majorana particles, there are also Dirac and Weyl particles. In the following their properties and the differences between them will be discussed, roughly along the lines of [3]. A crucial difference exists between massive and massless particles. Massless particles move at the speed of light while massive particles do not. Before neutrino oscillations indicated a non-vanishing mass, neutrinos were described by two-component complex spinors, or so-called Weyl spinors. But since Weyl particles have to be massless, the neutrino cannot be described as a Weyl particle anymore. Hence, the neutrino has to be either a Dirac or a Majorana particle.

Dirac particles have four states described by four basic spinors. Two of these are the left- and right-handed helicity states of a particle  $p_L, p_R$ , the other two the left- and right-handed helicity states of its antiparticle  $\bar{p}_L, \bar{p}_R$ . Note that the bar marks the antiparticles. Helicity  $h$  is the projection of the spin  $\vec{S}$  onto the direction of momentum  $\hat{p}$  [6, p.240]:

$$h = \vec{S} \cdot \hat{p} \quad \text{with} \quad \hat{p} \equiv \frac{\vec{p}}{|\vec{p}|}.$$

A massive particle cannot travel at the speed of light. This means that there can be a Lorentz boost to a system moving faster than the particle. We imagine a left-handed (LH) electron  $e_L$  moving in a certain direction with the speed  $v < c$ . An observer can be boosted along the particle into a system moving faster than the electron. From his perspective, the electron is seen as right-handed (RH). Two possible right-handed particles have been introduced, viz  $e_R$  and  $\bar{e}_R$ . We now want to distinguish which one the observer sees. Charge is a Lorentz invariant quantity. Since we started with an electron, which has negative charge, the particle in the boosted system must likewise have negative charge. Hence, it is the  $e_R$ . Here, the difference between a Dirac and a Weyl particle becomes obvious. A

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Weyl particle is massless, travels with velocity  $c$  and therefore an  $e_L$  can never be seen as an  $e_R$ .

Now we try the same, but this time using a neutrino instead of the electron. The neutrino is uncharged, so the term antiparticle is more or less not defined. The only thing left to use for the distinction between  $\nu_R$  and  $\bar{\nu}_R$  is the lepton number. According to the Dirac theory those two are two different particles. As said in the first section, Majorana proposed a theory where the lepton number conservation can be violated. Without conserving the lepton number there is no difference between  $\nu_R$  and  $\bar{\nu}_R$ . Thus, according to Majorana's proposal, the neutrino is its own antiparticle [3, pp.51].

## 2 Calculating a cross section

In this chapter it will be shown how a cross section is calculated. Therefore we start with deriving the Feynman rules in the following section. Feynman rules are a set of rules derived from the Lagrangian. These rules are used to calculate Feynman diagrams. Feynman diagrams are pictographic descriptions of the behaviour of subatomic particles. Afterwards a calculation example is conducted. For simplification the example does not consist of the entire Feynman diagram of  $0\nu\beta\beta$  [Fig.2]. Instead, we focus on the 'upper' part. There, a  $u$  quark and a  $\bar{d}$  quark collide and produce a  $W^+$  boson [Fig.3].

### 2.1 Feynman rules

The Lagrangian for a scalar field  $\varphi$  interacting with the Dirac field  $\psi$  is:

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi + \frac{1}{2} [(\partial\varphi)^2 - \mu^2\varphi^2] - \lambda\varphi^4 + f\varphi\bar{\psi}\psi.$$

A Dirac field is a fermionic field which describes spin 1/2 particles. A scalar field gives a certain scalar value to every point in space-time. The symbol  $\gamma^\mu$  denotes the gamma matrices, four matrices which solve the Dirac equation, the Schrödinger equation's analogue in relativistic particle physics. It applies  $\partial_\mu = \frac{\partial}{\partial x^\mu}$ ,  $\mu$  is the mass of the scalar particle  $\varphi$ ,  $m$  is the mass of the Dirac particle  $\psi$  and  $\lambda, f$  are coupling factors. Using this Lagrangian, the Feynman rules are:

1. Every momentum  $k$  of scalar particle  $\varphi$  is associated with the propagator  $i/(k^2 - m^2 + i\epsilon)$ .
2. To make sure we obey the conservation of momentum in the entire diagram we add the term  $(2\pi)^4 \cdot \delta^{(4)} \left( \sum_i k_i - \sum_j k_j \right)$ . Here  $k_i$  are the momenta of all initial particles and  $k_j$  are the momenta of all final particles.
3. Since the momenta of the inner lines are unknown, they are treated corresponding to ordinary perturbation theory. Hence, an integration will be carried out over all inner lines' momentum with  $d^4k/(2\pi)^4$ .

4. The propagator for initial (incoming) Dirac fermions with momentum  $k$  and spin  $s$  is  $u(k, s)$ , the one for final (outgoing) ones is  $\bar{u}(k, s)$ .
5. Initial antifermions are represented by  $\bar{v}(k, s)$  and final antifermions by  $v(k, s)$

For a massive vector field (bosons) interacting with a Dirac field the Lagrangian is [7, pp.129]:

$$\mathcal{L} = \bar{\psi} [i\gamma^\mu (\partial_\mu - ieA_\mu) - m] \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \mu^2 A_\mu A^\mu.$$

Here  $A$  denotes the field and  $F$  is the electromagnetic tensor. From this we obtain the boson propagator for external bosons  $\varepsilon_\mu(k, \lambda)$  with the momentum  $k$  and the polarisation  $\lambda$ .

Note that this is not a complete collection of Feynman rules, only those are listed which are needed in the following.

## 2.2 Calculating the matrix element of $u\bar{d} \rightarrow W^+$

After introducing the Feynman rules we now start with the calculation. As mentioned above, a  $u$  quark and a  $\bar{d}$  quark collide and produce a  $W^+$  boson. As explained e.g. in the book [7, p.128] we will start with drawing the diagram of the process we want to calculate and label all lines. In addition to the rules above, a representation for the vertex is also required. According to [8, p.714] the vertex for two fermions producing a boson is:

$$-i \frac{g}{\sqrt{2}} \gamma^\mu P_L V_{ff'},$$

where  $P_L = \frac{1-\gamma_5}{2}$  is the left-handed chirality projector and  $V_{ff'}$  is a matrix element. The index  $f$  denotes fermion. For our  $u$  and  $\bar{d}$  quarks this matrix element becomes  $V_{ud} = \cos\theta_C$ . The Cabbibo-angle  $\theta_C$  describes how quarks can change their flavour under the weak interaction [9]. The  $u$  is an initial fermion, thus it has the propagator  $u(p, s)$ . The  $\bar{d}$  is a initial antifermion which leads to  $\bar{v}(p', s')$

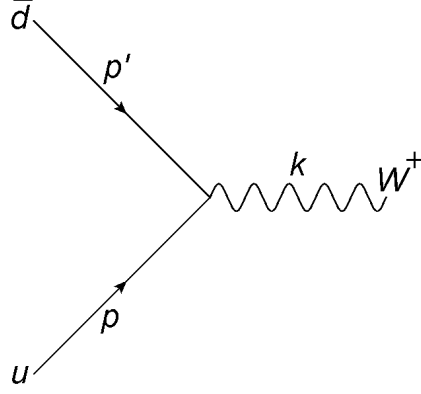


Figure 3: Feynman diagram of a  $u$  and a  $\bar{d}$  quark producing a  $W^+$  boson

as the propagator. The outgoing  $W^+$  boson is represented by  $\varepsilon_\mu^\dagger(k, r)$ . Combined this yields the matrix element:

$$\begin{aligned} \mathcal{M}(\bar{d}u \rightarrow W^+) &= \bar{v}(p', s') \left( -i \frac{g}{\sqrt{2}} \gamma^\mu P_L V_{ff'} \right) u(p, s) \varepsilon_\mu^\dagger(k, r) \\ &= -i \frac{g}{\sqrt{2}} \cos \theta_C \bar{v}(p', s') (\gamma^\mu P_L) u(p, s) \varepsilon_\mu^\dagger(k, r). \end{aligned} \quad (1)$$

In order to obtain the squared matrix element  $|\mathcal{M}|^2 = \mathcal{M}\mathcal{M}^\dagger$  the matrix element has to be hermitian conjugated:

$$\mathcal{M}^\dagger = i \frac{g}{\sqrt{2}} \cos \theta_C \cdot \left[ \varepsilon_\mu^\dagger(k, r) \right]^\dagger u(p, s)^\dagger (P_L)^\dagger (\gamma^\nu)^\dagger [\bar{v}(p', s')]^\dagger. \quad (2)$$

For further simplification we use a couple of relations [8, pp.131]:

$$(\gamma^0)^\dagger = \gamma^0 \quad (3)$$

$$(\gamma^\nu)^\dagger = (\gamma^0 \gamma^\nu \gamma^0) \quad (4)$$

$$\gamma^0 \gamma^0 = \gamma^5 \gamma^5 = \mathbb{1} \quad (5)$$

$$\bar{v}^\dagger = (v \gamma^0)^\dagger = (\gamma^0)^\dagger v = \gamma^0 v \quad (6)$$

$$(P_L)^\dagger = \left( \frac{\mathbb{1} - \gamma^5}{2} \right)^\dagger = \left( \frac{\mathbb{1}^\dagger - (\gamma^5)^\dagger}{2} \right) = \left( \frac{\mathbb{1} - \gamma^5}{2} \right) = P_L \quad (7)$$

$$\{\gamma^\mu, \gamma^5\} = \gamma^\mu \gamma^5 + \gamma^5 \gamma^\mu = 0 \quad (8)$$

$$P_L \gamma^0 = \frac{\gamma^0 - \gamma^5 \gamma^0}{2} = \frac{\gamma^0 + \gamma^0 \gamma^5}{2} = \gamma^0 P_R \quad (9)$$

$$(P_{R/L})^2 = \frac{(\mathbb{1} \pm \gamma^5)^2}{4} = \frac{\mathbb{1}^2 \pm 2\gamma^5 + (\gamma^5)^2}{4} = \frac{2\mathbb{1} \pm 2\gamma^5}{4} = P_{R/L} \quad (10)$$

Using at first the properties (4),(6) and (7) on equation (2) leads to:

$$\mathcal{M}^\dagger = i \frac{g}{\sqrt{2}} \cos \theta_C \cdot \varepsilon_\mu(k, r) \cdot u(p, s)^\dagger (P_L) (\gamma^0 \gamma^\nu \gamma^0) (\gamma^0)^\dagger v(p', s'). \quad (11)$$

Continuing with the identities of the gamma matrices (3), (5) and that the chirality projector  $P_L$  behaves like (9) we obtain:

$$\mathcal{M}^\dagger = i \frac{g}{\sqrt{2}} \cos \theta_C \cdot \varepsilon_\mu(k, r) \cdot \bar{u}(p, s) (P_R \gamma^\nu) v(p', s'). \quad (12)$$

The initial and the final spins are unknown, so we sum over all possibilities and average through adding the factor  $\frac{1}{4}$  (since by summing we count the spins of both quarks twice,  $\frac{1}{2}$ (for  $u$ )  $\cdot$   $\frac{1}{2}$ (for  $d$ ) =  $\frac{1}{4}$ ) to compute  $|\mathcal{M}|^2$ . The following relations will be used [8, p.132]:

$$\sum_s u(p, s) \bar{u}(p, s) = \not{p} + m \quad (13)$$

$$\sum_{s'} v(p', s') \bar{v}(p', s') = \not{p}' - m \quad (14)$$

$$\sum_r \varepsilon_\mu(k, r) \varepsilon_\nu^\dagger(k, r) = \left( \frac{k_\mu k_\nu}{m_W^2} - g_{\mu\nu} \right) \quad (15)$$

Here the  $\not{p}$  denotes the convenient Feynman slash notation  $\not{p} = \gamma^\mu p_\mu$ . The same averaging and summing is done with the three colors. Thus, we obtain the factor  $\frac{1}{3}$  (from:  $\sum_{a,b} \delta^a_b \cdot \frac{1}{3}(\text{for } u) \cdot \frac{1}{3}(\text{for } d) = 3 \cdot (\frac{1}{3})^2$ , where  $a, b$  are color indices). Putting all this together yields the squared matrix element:

$$|\mathcal{M}|^2 = \frac{1}{12} \sum_{s,s',r} |\mathcal{M}\mathcal{M}^\dagger| = \frac{1}{12} \sum_{s,s',r} \frac{g^2}{2} \cos^2 \theta_C \cdot \bar{v}(p', s') (\gamma^\mu P_L) u(p, s) \\ \times \varepsilon_\mu(k, r) \varepsilon_\nu^\dagger(k, r) \cdot \bar{u}(p, s) (P_R \gamma^\nu) v(p', s') \quad (16)$$

$$= \frac{g^2}{24} \cos^2 \theta_C \sum_r \varepsilon_\mu(k, r) \varepsilon_\nu^\dagger(k, r) \sum_{s'} v(p', s') \bar{v}(p', s') \cdot (\gamma^\mu P_L) \\ \times \sum_s u(p, s) \bar{u}(p, s) \cdot (P_R \gamma^\nu). \quad (17)$$

To make sure that the order within equation (16) is switched correctly, we would have to write down the spinor indices. This can be checked in [8, p.132]. Here we just carry out the sums into a trace using (13)-(15).

$$|\mathcal{M}|^2 = \frac{g^2}{24} \cos^2 \theta_C \cdot \left( \frac{k_\mu k_\nu}{m_W^2} - g_{\mu\nu} \right) \cdot \text{Tr} [(\not{p}' - m_{\bar{d}}) (\gamma^\mu P_L) (\not{p} + m_u) (P_R \gamma^\nu)]. \quad (18)$$

### 2.3 Traces and $\gamma$ matrices

In the next step, the quark masses  $m_d$  and  $m_u$  are set to zero since they are vanishingly small - a few MeV while the  $W^\pm$  boson weighs  $\sim 80$  GeV [4] - and we write out the slash notation  $\not{p} = \gamma^\mu p_\mu$ . Also the relations (7)-(10) are used again

in the following:

$$\begin{aligned}
\sum_{s,s',r} &= \frac{g^2}{24} \cos^2 \theta_C \left( \frac{k_\mu k_\nu}{m_W^2} - g_{\mu\nu} \right) Tr \left[ (\not{p}' - m_{\bar{d}}) (\gamma^\mu P_L) (\not{p} + m_u) (P_R \gamma^\nu) \right] \\
&= \frac{g^2}{24} \cos^2 \theta_C \left( \frac{k_\mu k_\nu}{m_W^2} - g_{\mu\nu} \right) Tr \left[ (\not{p}') (\gamma^\mu P_L) (\not{p}) (P_R \gamma^\nu) \right] \\
&= \frac{g^2}{24} \cos^2 \theta_C \left( \frac{k_\mu k_\nu}{m_W^2} - g_{\mu\nu} \right) Tr \left[ (\gamma^\alpha p_\alpha) (\gamma^\mu P_L) (\gamma^\beta p_\beta) (P_R \gamma^\nu) \right] \\
&= \frac{g^2}{24} \cos^2 \theta_C \left( \frac{k_\mu k_\nu}{m_W^2} - g_{\mu\nu} \right) (p'_\alpha p_\beta) Tr \left[ \gamma^\alpha \gamma^\mu P_L \gamma^\beta P_R \gamma^\nu \right] \\
&= \frac{g^2}{24} \cos^2 \theta_C \left( \frac{k_\mu k_\nu}{m_W^2} - g_{\mu\nu} \right) (p'_\alpha p_\beta) Tr \left[ \gamma^\alpha \gamma^\mu \gamma^\beta P_R P_R \gamma^\nu \right] \\
&= \frac{g^2}{24} \cos^2 \theta_C \left( \frac{k_\mu k_\nu}{m_W^2} - g_{\mu\nu} \right) (p'_\alpha p_\beta) Tr \left[ \gamma^\alpha \gamma^\mu \gamma^\beta P_R \gamma^\nu \right] \\
&= \frac{g^2}{24} \cos^2 \theta_C \left( \frac{k_\mu k_\nu}{m_W^2} - g_{\mu\nu} \right) (p'_\alpha p_\beta) Tr \left[ \gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu P_L \right] \\
&= \frac{g^2}{24} \cos^2 \theta_C \left( \frac{k_\mu k_\nu}{m_W^2} - g_{\mu\nu} \right) (p'_\alpha p_\beta) Tr \left[ \gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu \frac{1 - \gamma^5}{2} \right] \\
&= \frac{g^2}{24} \cos^2 \theta_C \left( \frac{k_\mu k_\nu}{m_W^2} - g_{\mu\nu} \right) \left( \frac{p'_\alpha p_\beta}{2} \right) \left( Tr \left[ \gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu \right] + Tr \left[ \gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu \gamma^5 \right] \right).
\end{aligned} \tag{19}$$

For the traces of the gamma matrices we have the properties [8, p.134]

$$Tr \left[ \gamma^\alpha \gamma^\beta \gamma^\delta \gamma^\epsilon \right] = 4 \left( g^{\alpha\beta} g^{\delta\epsilon} - g^{\alpha\delta} g^{\beta\epsilon} + g^{\alpha\epsilon} g^{\beta\delta} \right) \tag{20}$$

$$Tr \left[ \gamma^\alpha \gamma^\beta \gamma^\delta \gamma^\epsilon \gamma^5 \right] = -4i \varepsilon_{\alpha\beta\delta\epsilon}. \tag{21}$$

The last term of equation (19) disappears as shown in the following, using trace property (21). The  $\varepsilon$  here is called Levi-Civita symbol. It is an - in this case four-dimensional - antisymmetric pseudo tensor. The Levi-Civita symbol is 1 for every even index permutation,  $-1$  for every odd permutation and becomes zero if an index is repeated. Since  $k_\mu k_\nu = k_\nu k_\mu$  and  $g_{\mu\nu} = g_{\nu\mu}$  apply, and thus are



symmetric, the following yields zero:

$$k_\mu k_\nu p'_\alpha p_\beta \varepsilon^{\alpha\mu\beta\nu} = 0 \quad (22)$$

$$g_{\mu\nu} p'_\alpha p_\beta \varepsilon^{\alpha\mu\beta\nu} = 0 \quad (23)$$

Inserting trace property (20) in the non-disappearing first part of equation (19) leads to:

$$|\mathcal{M}|^2 = \frac{g^2}{24} \cos^2 \theta_C \left( \frac{k_\mu k_\nu}{m_W^2} - g_{\mu\nu} \right) \left( \frac{p'_\alpha p_\beta}{2} \right) 4 (g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\beta} g_{\mu\nu} + g_{\alpha\nu} g_{\mu\beta}) \quad (24)$$

$$= \frac{g^2}{12} \cos^2 \theta_C \left( \frac{k_\mu k_\nu}{m_W^2} - g_{\mu\nu} \right) (p'^\mu p^\nu - (p' \cdot p) g_{\mu\nu} + p'^\nu p^\mu) \quad (25)$$

$$= \frac{g^2}{12} \frac{\cos^2 \theta_C}{m_W^2} (2(k \cdot p')(k \cdot p) - (p' \cdot p)k^2 + 2(p' \cdot p)m_W^2) \quad (26)$$

We assume the  $W^+$  boson is on shell. This means that we emanate the particle to be real and therefore it applies that the squared four-momentum vector is equal to the squared mass, see [7, p.56]. Hence, we set  $k^2 = m_W^2 = g^{\mu\nu} k_\mu k_\nu$  and already cancelled the quark masses to zero. For the squared four momenta of the quarks this means:  $p^2 = p'^2 = 0$ . From momentum conservation we obtain  $p + p' = k$ . Using this it can be concluded that:

$$(p' + p)^2 = (p' + p)_\mu (p' + p)_\nu g^{\mu\nu} = 2p' \cdot p = k^2 = m_W^2$$

$$\rightarrow p' \cdot p = 1/2 m_W^2 \quad (27)$$

$$p^2 = 0 = (k - p')^2 = k^2 - 2(k \cdot p') + p'^2$$

$$\rightarrow k \cdot p' = k \cdot p = 1/2 m_W^2. \quad (28)$$

Thereby we finally obtain an expression only depending on constant factors:

$$|\mathcal{M}|^2 = \frac{g^2}{12} \cdot \cos^2 \theta_C \cdot m_W^2. \quad (29)$$

So far, this result can be confirmed, see e.g. [10].

## 2.4 Partonic cross section $\hat{\sigma}$ ( $u\bar{d} \rightarrow W^+$ )

Parton is an old expression for the constituents of an hadron, now known as quark. Thus, a cross section calculation on the partonic level means that we pretend to collide two solitary quarks instead of the hadrons - protons in our case - we collide in experiments. For the transition into an experimentally verifiable cross section we will need parton distribution functions. We will return to this topic later.

In order to derive a cross section from (29), the following general formula from [8, p.106] is used:

$$d\hat{\sigma} = \frac{1}{2E_A 2E_B |v_A - v_B|} |\mathcal{M}|^2 (2\pi)^4 \underbrace{\left( \prod_f \frac{d^3 p_f}{(2\pi)^3} \cdot \frac{1}{2E_f} \right)}_{\Gamma} \delta^{(4)}(p_A + p_B - \sum p_f). \quad (30)$$

The part denoted with  $\Gamma$  is the phase space factor. The phase space is a multidimensional space which represents all states the system can attend. The factor is needed to average them out. The indices  $A, B$  indicate energy and momentum of the initial particles  $u$  and  $\bar{d}$  with their momenta  $p$  and  $p'$ . The  $f$ 's symbolize all final particles. Since we have only one final particle, viz the W boson, equation (30) simplifies to:

$$d\hat{\sigma} = \frac{1}{2E_p 2E_{p'} |v_p - v_{p'}|} \left( \frac{d^3 k}{(2\pi)^3} \cdot \frac{1}{2E_W} \right) |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(p + p' - k). \quad (31)$$

From [8, p.106] we derive:

$$\frac{1}{2E_p 2E_{p'} |v_p - v_{p'}|} = \frac{1}{4 |E_{p'} p - E_p p'|} \quad \text{with} \quad v_i \equiv \frac{\vec{p}_i}{E_i}.$$

In our case - using now  $p_u$  and  $p_d$  for the four momenta of the incoming quarks - this yields [11, p.64]:

$$\frac{1}{4 |E_d p_u - E_u p_d|} = \frac{1}{4 \sqrt{(p_u \cdot p_d)^2 - m_u^2 m_d^2}} = \frac{1}{4 \sqrt{(p_u \cdot p_d)^2}}. \quad (32)$$

We continue with inserting  $p_u \cdot p_d = \frac{1}{2}(p_u + p_d)^2 = \frac{1}{2}k^2 = \frac{1}{2}m_W^2$  in (32) and obtain:

$$\frac{1}{4 |E_d \vec{p}_u - E_u \vec{p}_d|} = \frac{1}{2m_W^2}. \quad (33)$$

We keep this in mind and address the delta function in (31). We start with splitting up the four-vector delta function:

$$\delta^{(4)}(p + p' - k) = \delta^{(0)}(p^0 + p'^0 - k^0) \cdot \delta^{(3)}(\vec{p} + \vec{p}' - \vec{k}). \quad (34)$$

From  $\delta^{(3)}(\vec{p} + \vec{p}' - \vec{k})$  we sustain that:

$$k^0 = E_k = \sqrt{\vec{k}^2 + m_W^2} = \sqrt{(\vec{p}_u + \vec{p}_d)^2 + m_W^2} = m_W^2. \quad (35)$$

Here we assumed the center of mass frame so that the initial momenta have opposite signs and cancel each other out. We also know that in the center of mass frame it is  $p_u^0 + p_d^0 = E_u + E_d \equiv 2E \equiv E_{cm} = \sqrt{s}$ . Using this and equation (35) we obtain for  $\delta^{(0)}(p^0 + p'^0 - k^0)$ :

$$\delta^{(0)}(p^0 + p'^0 - k^0) = \delta^{(0)}(p_u^0 + p_d^0 - k^0) = \delta(\sqrt{s} - m_W). \quad (36)$$

We insert the simplifications (33), (36) and the result for the matrix element from (29) in equation (31):

$$\begin{aligned} d\hat{\sigma} &= \frac{1}{2m_W^2} \left( \frac{d^3k}{(2\pi)^3} \cdot \frac{1}{2E_W} \right) \frac{g^2}{12} \cos^2 \theta_C \cdot m_W^2 (2\pi)^4 \delta^{(3)}(\vec{p} + \vec{p}' - \vec{k}) \delta^{(0)}(p_u^0 + p_d^0 - k^0) \\ &= \pi \frac{g^2}{12} \cos^2 \theta_C d^3k \frac{1}{2E_W} \delta^{(3)}(\vec{p} + \vec{p}' - \vec{k}) \delta^{(0)}(p_u^0 + p_d^0 - k^0). \end{aligned} \quad (37)$$

In order to obtain an expression for the cross section  $\sigma$  we integrate over  $d^3k$ :

$$\begin{aligned} \hat{\sigma}(u \bar{d} \rightarrow W^+) &= \frac{g^2}{12} \pi \cdot \cos^2 \theta_C \int d^3k \frac{1}{2E_W} \delta^{(3)}(\vec{p} + \vec{p}' - \vec{k}) \delta^{(0)}(p_u^0 + p_d^0 - k^0) \\ &= \frac{g^2}{12} \pi \cdot \cos^2 \theta_C \frac{1}{2m_W} \delta(\sqrt{s} - m_W) \end{aligned} \quad (38)$$

$$= \frac{g^2}{12} \pi \cdot \cos^2 \theta_C \delta(s - m_W^2). \quad (39)$$

Using the following relation for  $\delta$ -functions [6], we can transform (38) into (39):

$$\delta(g(x)) = \sum_i \frac{\delta(x - x_i)}{|g'(x_i)|} \text{ with } g(x_i) = 0 \text{ and } g'(x_i) \neq 0, \quad (40)$$

using  $g(x) = \sqrt{s} - m_W$ ,  $g(x) = 0 \rightarrow \sqrt{s} - m_W = 0 \rightarrow s = m_W^2$  and  $g'(s) = \frac{1}{2\sqrt{s}}$  we obtain:

$$\delta(\sqrt{s} - m_W) = \sum_i \frac{\delta(s - s_i)}{g'(s_i)} = 2 m_W \delta(s - m_W^2) \quad (41)$$

The result for  $\hat{\sigma}$  in (39) can be validated e.g. in the ‘pink book’ [12, p.313].

## 2.5 Analytical and Numerical results for $\sigma (u\bar{d} \rightarrow W^+)$

The values we use for the calculation should match the values MadGraph uses as well as current experimental limits. We will use:

$$\begin{aligned} \cos \theta_C &= 0.97425 \pm 0.00022 & [4] \\ g^2 &= 4\sqrt{2}G_F m_W^2 \\ G_F &= 1.16639 \times 10^{-5} \text{ GeV}^{-2} \\ m_W &= (80.385(15) \pm 1.9 \times 10^5) \text{ GeV} & [4] \end{aligned}$$

This yields:

$$\begin{aligned} \hat{\sigma} (u\bar{d} \rightarrow W^+) &= \frac{\sqrt{2}}{3} \pi \cos^2 \theta_C G_F m_W^2 \delta(s - m_W^2) \\ &= 0.105936 \delta(s - m_W^2). \end{aligned} \quad (42)$$

It is obvious that this cannot be the final result since it still has a  $\delta$ -function. This is due to the fact that - as said in the previous section - we ‘collided’ solitary quarks in this calculation. Thus, we now use the mentioned parton distribution functions. A parton distribution function (pdf) specifies the probability of finding a parton of a certain flavour carrying a fraction  $x$  of the hadron’s momentum with

$Q$  as the energy of the hadron's interaction. There are different pdf's depending on the parton. For more details on pdf's see e.g. [13] or [8]. In order to obtain the full cross section, we need to convolve (42) with these parton distribution functions for  $u$  and  $d$  quarks, see e.g. [12, pp.301]. :

$$\begin{aligned}\sigma (u\bar{d} \rightarrow W^+) &= \int_0^1 dx_1 \int_0^1 dx_2 f_u(x_1) f_{\bar{d}}(x_2) \hat{\sigma} (u\bar{d} \rightarrow W^+) \\ &= \int_0^1 dx_1 \int_0^1 dx_2 f_u(x_1) f_{\bar{d}}(x_2) 0.105936 \delta(s - m_W^2).\end{aligned}\quad (43)$$

We use the substitution  $s = x_1 x_2 s'$  [12, p.303], where  $s$  is the squared energy in the  $\delta$ -function above,  $Q = s'$  is the squared energy for the complete collision, here  $s' = (14 \text{ TeV})^2$  and  $x_1, x_2$  describe the momentum distribution of the quarks.

$$\begin{aligned}\sigma (u\bar{d} \rightarrow W^+) &= 0.105936 \int_0^1 dx_1 \int_0^1 dx_2 f_u(x_1) f_{\bar{d}}(x_2) \delta(x_1 x_2 s' - m_W^2) \\ &= 0.105936 \int_0^1 dx_1 \int_0^1 dx_2 f_u(x_1) f_{\bar{d}}(x_2) \frac{1}{x_2 s'} \delta\left(x_1 - \frac{m_W^2}{x_2 s'}\right) \\ &= 0.105936 \int_0^1 dx_2 f_u\left(\frac{m_W^2}{x_2 s'}\right) f_{\bar{d}}(x_2) \frac{1}{x_2 s'}.\end{aligned}\quad (44)$$

Until now we used natural units ( $c = \hbar = 1$ ). For the conversion between the natural units and SI units the following applies:

$$\frac{1}{\text{GeV}^2} = \frac{\hbar^2 c^2}{\text{GeV}^2} = \left(\frac{6.582 \text{ GeV s} \cdot 2.998 \frac{\text{m}}{\text{s}}}{\text{GeV}} \times 10^{-17}\right)^2 = 3.893 \times 10^{-32} \text{ m}^2, \quad (45)$$

using  $\hbar = 6.582 \times 10^{-25} \text{ GeV s}$  and  $c = 2.998 \times 10^8 \frac{\text{m}}{\text{s}}$  [4].

The second integration can be conducted with Mathematica and external pdfs, here the MSTW [14], see appendix C. Finally, we obtain the cross section:

$$\sigma (u\bar{d} \rightarrow W^+) = 4.0349 \times 10^4 \text{ pb.} \quad (46)$$

Since calculating a more difficult cross section by hand is fairly unhandy, we want to use MadGraph to calculate the  $0\nu\beta\beta$  cross section. An explanation how to

use MadGraph can be found in appendix B. MadGraph generates the  $W$  boson production diagrams with:

```
> generate u d~ > w+
```

and then calculates the cross section:

$$\sigma_{MG}(u\bar{d} \rightarrow W^+) = 3.959 \times 10^4 \pm 53.97 \text{ pb.} \quad (47)$$

In the output directory in the file `Source/PDF/pdf_list.txt`, all pdf's MadGraph can use are denoted. Per default and in our calculation MadGraph uses the parton distribution functions CTEQ6L from [15] for this calculation. Using a different pdf slightly changes the cross section's value.

Comparing the results (46) and (47) we see that the results are not totally compatible with each other within the margin of error but close and that they are of the same order. This is to be expected since we used different pdfs and this is satisfactory for our purpose.

### 3 Left-right symmetric model

In experiments it has been observed that the boson for the charged weak interaction  $W^\pm$  only couples to LH particles and RH antiparticles. Thus, this was imposed in the standard model without further theoretical motivation [8, p.703]. In the thought experiment in section 1.3, we used additional RH particles and LH antiparticles. To expand the particle content, we will use a minimal left-right symmetric model (LRSM) to extend the SM. At higher energies than currently investigated, the LRSM obeys its own gauge symmetry (see appendix A.3). Hence, the SM gauge symmetry (see A.2) is broken and gives different masses to the LH and RH gauge bosons  $W^\pm$ . For lower energies, the LRSM becomes equal to the SM.

We still do not know where the neutrino mass has its origin. Since the mass is of an order far below all other known particle masses - the neutrinos mass is below one eV while e.g. the electron mass is 511 eV [4] - its source may be something else than the Higgs mechanism. One solution is the seesaw theory. In the seesaw theory, the light neutrino gains a small Majorana mass through a heavy counterpart via Yukawa coupling. The name seesaw refers to the eigenvalues. The general neutrino mass matrix with Majorana mass  $M$  and Dirac mass  $m$  is of the form:

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \quad \text{with } M \gg m \quad \rightarrow \lambda_{\pm} = \frac{M \pm \sqrt{M^2 - 4m^2}}{2}$$

If one of the eigenvalues  $\lambda_{\pm}$  goes up, the other goes down, and vice versa, like on a seesaw.

There are three different kinds of seesaw types, depending on how the heavy counterpart that gives mass to the neutrino is implemented. Type I uses gauge-singlets, Type II gauge-triplets - both under  $SU(2)$  - and Type III adds the counterpart itself in form of a fermionic triplet. The latter will be used in this thesis because it is the one which is most likely to be observable in an LHC process [16].

### 3.1 Type III seesaw

In this thesis we will use a Type III seesaw model and calculate a  $0\nu\beta\beta$  cross section with MadGraph 5 [17]. The model file that will be used for the MadGraph calculation originated from [18]. Here a minimal realisation of the Type III seesaw is implemented. This is done in the form of adding fermionic  $SU(2)$  triplets, consisting of two heavy charged Dirac leptons  $\Sigma^\pm$  and a heavy neutral Majorana lepton  $\Sigma^0$ . In addition to the SM we add these triplets to gain neutrinos with non-vanishing masses. The extra fermions  $\Sigma$  have zero hypercharge  $Y$ . Their interaction beyond the SM is denoted by the Lagrangian [19]:

$$\mathcal{L} = Tr [\bar{\Sigma} i \not{D} \Sigma] - \frac{1}{2} [\bar{\Sigma} M_\Sigma \Sigma^c + \bar{\Sigma}^c M_\Sigma^* \Sigma] - \tilde{\phi}^\dagger \bar{\Sigma} \sqrt{2} Y_\Sigma L - \bar{L} \sqrt{2} Y_\Sigma^\dagger \Sigma \tilde{\phi}, \quad (48)$$

with the lepton doublets  $L \equiv (\nu, l)^T$ , the scalar field  $\phi \equiv (\phi^+, \phi^0)^T \equiv (\phi^+, (v + H + i\eta)/\sqrt{2})^T$ ,  $\tilde{\phi} \equiv i\tau_2 \phi^*$ ,  $\Sigma^C \equiv C \bar{\Sigma}^T$  with the charge conjugation operator  $C$  and with, for each fermionic triplet,

$$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix}, \quad \Sigma^C = \begin{pmatrix} \Sigma^{0c}/\sqrt{2} & \Sigma^{-c} \\ \Sigma^{+c} & -\Sigma^{0c}/\sqrt{2} \end{pmatrix},$$

$$D_\mu = \partial_\mu - i\sqrt{2}g \begin{pmatrix} W_\mu^3/\sqrt{2} & W_\mu^+ \\ W_\mu^- & -W_\mu^3/\sqrt{2} \end{pmatrix}. \quad (49)$$

For a more detailed description of the model see [19]. Interesting for the purpose of this thesis is the mass of the heavy neutral Majorana lepton  $\Sigma^0$ . We use a simplified model with just one triplet [19, p.4]. With this assumption the Yukawa couplings matrix becomes a  $1 \times 3$  vector:  $Y_\Sigma = (Y_{\Sigma_e} \ Y_{\Sigma_\mu} \ Y_{\Sigma_\tau})$ . The mass matrix  $M_\Sigma$  reduces to a scalar. We also assume that all parameters are real. This means we do not consider the phase of the Yukawa couplings or those of the PMNS matrix. The PMNS (Pontecorvo–Maki–Nakagawa–Sakata) matrix is the mixing matrix equivalent for the flavour and mass eigenstates mixing of neutrinos, for more see e.g. [20].



## 4 Limits and Experiments

The half-life of the neutrinoless double beta decay and the neutrino masses are interdependent according to [16]:

$$\frac{1}{T_{0\nu}^{1/2}} = G^{0\nu}(Q, Z) |\mathcal{M}^{0\nu}|^2 |\eta|^2, \quad (50)$$

with  $G^{0\nu}(Q, Z)$  the phase space factor and  $\mathcal{M}^{0\nu}$  the Nuclear Matrix Element (NME).  $\eta$  is a dimensionless particle physics parameter, defined as [16]:

$$\eta = \left( \frac{m_W}{m_{W_R}} \right)^4 \frac{m_p}{\langle m_{\Sigma_0} \rangle}, \quad (51)$$

with the proton mass  $m_p$ , the mass of the common (LH)  $W^\pm$  boson  $m_W$ , the mass of the RH  $W$  boson  $m_{W_R}$ , added through the LRSM, and the mass of the heavy neutrino  $m_{\Sigma_0}$ . As seen in (50), the mass of the heavy neutrino corresponds to its decay width. A recent and improved calculation for the phase space factors of  $2\nu\beta\beta$  and  $0\nu\beta\beta$  can be found in [21] or [22]. Current values for Nuclear Matrix Elements are presented in [23]. In the next subsection, we will discuss the various experiments on double beta decay half-lives and present their results. In the subsequent subsection, we discuss the mass of the heavy charged neutrino.

### 4.1 Experimental limits on double beta decay half-life

There are several experiments currently taking data on double beta decay. The dominant difficulty is the rareness of both kinds of double beta decay which can easily hide in other radiation. Therefore the general setup is very similar among most experiments. The elements used in  $0\nu\beta\beta$  experiments are:

$^{48}\text{Ca}$ ,  $^{76}\text{Ge}$ ,  $^{82}\text{Se}$ ,  $^{96}\text{Zr}$ ,  $^{100}\text{Mo}$ ,  $^{100}\text{Mo} - ^{100}\text{Ru}$  ( $0_1^+$ ),  $^{116}\text{Cd}$ ,  $^{130}\text{Te}$ ,  $^{150}\text{Nd}$ ,  $^{150}\text{Nd} - ^{150}\text{Sm}(0_1^+)$  and  $^{238}\text{U}$ .

The sample of one of these materials is located in a large water tank. The tank is heavily shielded with lead against background radiation. To minimize the chance of radiation from space coming through, most of the laboratories are underground.

Isotope	$T_{1/2}(2\nu)$ , yr	$M^{2\nu}$
$^{48}\text{Ca}$	$4.4^{+0.6}_{-0.5} \cdot 10^{19}$	$0.0238^{+0.0015}_{-0.0017}$
$^{76}\text{Ge}$	$(1.5 \pm 0.1) \cdot 10^{21}$	$0.0716^{+0.0025}_{-0.0023}$
$^{82}\text{Se}$	$(0.92 \pm 0.07) \cdot 10^{20}$	$0.0503^{+0.0020}_{-0.0018}$
$^{96}\text{Zr}$	$(2.3 \pm 0.2) \cdot 10^{19}$	$0.0491^{+0.0023}_{-0.0020}$
$^{100}\text{Mo}$	$(7.1 \pm 0.4) \cdot 10^{18}$	$0.1258^{+0.0037}_{-0.0034}$
$^{100}\text{Mo}-^{100}\text{Ru}(0_1^+)$	$5.9^{+0.8}_{-0.6} \cdot 10^{20}$	$0.1017^{+0.0056}_{-0.0063}$
$^{116}\text{Cd}$	$(2.8 \pm 0.2) \cdot 10^{19}$	$0.0695^{+0.0025}_{-0.0024}$
$^{128}\text{Te}$	$(1.9 \pm 0.4) \cdot 10^{24}$	$0.0249^{+0.0031}_{-0.0023}$
$^{130}\text{Te}$	$(6.8^{+1.2}_{-1.1}) \cdot 10^{20}$	$0.0175^{+0.0016}_{-0.0014}$
$^{150}\text{Nd}$	$(8.2 \pm 0.9) \cdot 10^{18}$	$0.0320^{+0.0018}_{-0.0017}$
$^{150}\text{Nd}-^{150}\text{Sm}(0_1^+)$	$1.33^{+0.45}_{-0.26} \cdot 10^{20}$	$0.0250^{+0.0029}_{-0.0034}$
$^{238}\text{U}$	$(2.0 \pm 0.6) \cdot 10^{21}$	$0.0271^{+0.0053}_{-0.0033}$
$^{130}\text{Ba}$ ; ECEC( $2\nu$ )	$(2.2 \pm 0.5) \cdot 10^{21}$	$0.105^{+0.014}_{-0.010}$

Table 1: Half-life and NME values for two-neutrino double beta decay  $2\nu\beta\beta$ . Table taken from [24].

Also the perturbation from inside may be reduced. Hence, the material in use has to be highly purified and is enriched with the double beta decaying isotope. The half-life measurement is performed as follows: observed events are tested on whether they are a double beta decay and are counted if they are. From the decay count and the mass of material the half-life can be calculated. In the KamLAND-Zen experiment, for example, an event rate of  $80.9 \pm 0.7$  (ton day) $^{-1}$  in 129 kg of  $^{136}\text{Xe}$  was measured [25, p.5]. This leads to a half-life of  $T_{1/2}^{2\nu} = 2.38 \pm 0.02$  (stat)  $\pm 0.14$  (sys)  $\times 10^{21}$  yr. In [24], all available experimental results up to March 2010 were analyzed and summed up within their compatibilities. The resulting half-lives with errors and the NMEs based on these

half-lives, are presented in table 1. Even more recent values from GERDA can be found in [26].

In 2001 a part of the Heidelberg-Moscow collaboration claimed the actual observation of a neutrinoless double beta decay [27]. The claim was disputed by the rest of the collaboration as well as other physicists and can be safely ignored [28]. From the observation of zero events, one can at least derive a lower limit for the half-life. For example, no observation of a decay in the 129 kg  $^{136}\text{Xe}$  in KamLAND-Zen, results in a half life of minimum  $T_{1/2}^{0\nu} > 5.7 \times 10^{24}$  yr [25]. The most recent limit from GERDA [26] is the minimum half life  $T_{1/2}^{0\nu} > 3 \times 10^{25}$  yr.

## 4.2 Limits on the neutrino masses

In this section we want to narrow down the mass of the heavy neutral neutrino  $m_{\Sigma_0}$ . Lower bounds on this mass have been determined from experiments e.g. by the CMS collaboration [29]. The data used was recorded at the LHC in 2011, it corresponds to an integrated luminosity of  $4.9 \text{ fb}^{-1}$  from proton-proton collisions at  $\sqrt{s} = 7 \text{ TeV}$ . Using our Type III seesaw model the collaboration concluded lower limits ranging from 180 – 200 GeV. Hence, we obtain  $m_{\Sigma_0} > 180 \text{ GeV}$ . These bound holds if one or more of the mixing parameters  $V_e, V_\mu, V_\tau$  is larger than  $10^{-6}$  [29]. We will keep this in mind for the next chapter.

It is also possible to calculate a limit using equation (50) and the numbers provided in the papers stated at the beginning of this section. Permuting (50) yields:

$$\langle m_{\Sigma_0} \rangle = \sqrt{T_{0\nu}^{1/2} G^{0\nu} \mathcal{M}^{0\nu}} \cdot \left( \frac{m_W}{m_{W_R}} \right)^4 m_p \quad (52)$$

For the calculation we will use the experimental values for Germanium-76 since the most recent half life values have been obtained for this element.

$$T_{0\nu}^{1/2} > 3.0 \times 10^{25} \text{ yr} \quad [26]$$

$$G^{0\nu} = 2.34 \times 10^{-15} \text{ yr}^{-1} \quad [22]$$

$$\mathcal{M}^{0\nu} = 32.6 - 48.1 \quad [23]$$

$$m_W = 80.385 \text{ GeV} \quad [4]$$

$$m_p = 0.938 \text{ GeV} \quad [4]$$

$$m_{W_R} < 2.5 \times 10^3 \text{ GeV} \quad [30]$$

These numbers provide the lower bound  $m_{\Sigma_0} > 125,348 \text{ GeV}$ . The bound on the RH boson mass  $m_{W_R}$  is the lowest possible bound obtained from LHC results. Lower masses have been excluded since otherwise it would have been already observed. We choose the RH boson mass to meet this lower bound in order to obtain the strictest limit on the heavy neutrino mass  $m_{\Sigma_0}$ . However, this limit is still lower than the one from the CMS data. We will discuss the consequences of different neutrino masses on the result of the cross section calculation in the next chapter.

## 5 Cross section of $pp \rightarrow \Sigma_0 \rightarrow jje^-e^-$

In this section, a cross section of neutrinoless double beta decay will be calculated using a Type III seesaw model [19] with a heavy neutrino  $\Sigma_0$  in MadGraph5 [17].

In all the runs, MadGraph calculates the cross section with a total of 10000 events and the LHC center of mass energy  $\sqrt{s} = 14$  TeV. This value can be changed in the run card but we will not do so. The collider mode e.g. pp,  $e^+e^-$  can also be changed there. The equivalent of the process  $pp \rightarrow \Sigma_0 \rightarrow jje^-e^-$  as a MadGraph input is:

```
> generate p p > tr0 > j j e- e-
```

An unmodified run of the process using the Seesaw Model from [19] leads to a zero cross section. This is due to the fact that the parameters `yme` and `Ve` have been set to zero. Both parameters contribute to the value of a coupling which should be non-zero in order to obtain a cross section. The parameter `yme` - denoted as ‘electron yukawa mass’ in the .fr file - corresponds to the electron mass in GeV, viz  $m_e = 5.11 \times 10^{-4}$  [4]. Hence, we set `yme` = 5.11e-4. Concerning the mixing parameter `Ve` - the ‘electron mixing’ according to the .fr file - bounds with respect to the model are presented in [19, p.10]:

$$|V_e| < 5.5 \times 10^{-2}$$

$$|V_\mu| < 6.3 \times 10^{-2}$$

$$|V_\tau| < 6.3 \times 10^{-2}$$

$$|V_e V_\mu| < 1.7 \times 10^{-7} \tag{53}$$

$$|V_e V_\tau| < 4.2 \times 10^{-4} \tag{54}$$

$$|V_\mu V_\tau| < 4.9 \times 10^{-4} \tag{55}$$

For now, the value of the cross section depends mostly on the parameter `Ve`. In order to obtain the maximum event count, we vary its value and then change `Vμ` and `Vτ` accordingly. The largest cross section arises from the maximum value for `Ve`. Thus, we set `Ve` = 5.5e-2. From this value and (53)-(55) the bounds on the

other parameters are determined as follows:

$$V_\mu < 3.09 \times 10^{-6}$$

$$V_\tau < 7.64 \times 10^{-3}$$

These assumptions yield a cross section of  $1.731 \times 10^{-5} \pm 6.401 \times 10^{-8}$  pb. The number of events  $\#$  to expect with given cross section  $\sigma$  and luminosity  $L$  can be calculated via  $\# = L \cdot \sigma$ . Thus, our cross section would correspond to 0.4 events in the LHC's record data collection of  $L = 23 \text{ fb}^{-1}$  in 2012 <sup>1</sup>.

The remaining parameter we need to vary is the mass of the heavy uncharged neutrino. From LHC data using the same boundary conditions as we do, a lower limit of 180-210 GeV was derived [29]. As mentioned (before), this bound is only valid if at least one of the mixing parameter is larger than  $10^{-6}$  which is indeed the case. Thus, from the calculation in the previous section we obtained the limit  $m_{\Sigma_0} > 125,348 \text{ GeV}$  which is a little below the CMS limit. In the original file, the mass is set to `mtr0 = 100.8 GeV`.

Changing the mass to `mtr0 = 210 GeV`, in order to meet the limit given by the CMS data, yields the cross section of  $4.036 \times 10^{-5} \pm 1.754 \times 10^{-7}$  pb. This corresponds to 1 event per year.

There was no upper limit to be found so we could increase the mass as far as we want to. Doing so however just yields smaller cross sections. Thus, we obtained the largest cross section with  $m_{\Sigma_0} = 210 \text{ GeV}$  and  $V_e = 5.5 \times 10^{-2}$ .

---

<sup>1</sup>[http://lpc.web.cern.ch/lpc/lumiplots\\_2012.htm](http://lpc.web.cern.ch/lpc/lumiplots_2012.htm)

## 6 Conclusion

We aimed to give a number of  $0\nu\beta\beta$  events that would be visible at the LHC using two proton beams with a center of mass energy of  $\sqrt{s}=14$  TeV. Therefore we assumed a Type III seesaw model. Within the bounds from the model and the bounds yielded from recent experimental results, we varied the parameters to obtain a most optimal number of events. With these values we obtained a cross section of  $4.036 \times 10^{-5} \pm 1.754 \times 10^{-7}$  pb. Assuming the LHC has a collection of data of  $23\text{fb}^{-1}$ , this corresponds to one observable event per year. This result is not significant.

To obtain a more meaningful result we could await an increase in the LHC's luminosity, a project which would take several decades. Another point of discussion are the assumptions made in the course of this thesis. Especially replacing the Higgs mechanism with a seesaw theory is not necessarily the correct approach. In addition, the type III seesaw is a very specific model; we added a triplet of new particles and neglected the Yukawa coupling phases. And assuming the model holds, there are still pretty large uncertainties in values used for the calculation, for example in the NME or the mass of the heavy neutrino.

As there is a multitude of conceivable models - see [31, table 14] for an overview - the amount of time one wants to spend on calculating cross sections can be chosen at will. Hence, a more promising project could be improving current  $0\nu\beta\beta$  experiments in order to actually observe the decay. This would prove the Majorana nature of the neutrino. Nonetheless, this would still leave us with the need for a proper theoretical description of the whole process.

# Appendix

## A Gauge Theory and Symmetries

Emmy Noether's first theorem states that every continuous symmetry of a physical system has a conservation law which corresponds to it [32]. This awareness is used in the Lagrangian formulation. The knowledge of a certain conservation law of a physical system, combined with the Hamiltonian principle, reduces the degrees of freedom in this system and simplifies the construction of the Lagrangian. The other way around with a given Lagrangian, it is easy to identify the conserved quantities from the invariance of the associated currents under transformations of the action. For a deeper insight in this topic, use for example [33]. These symmetries under transformation can be 'local' or 'global'. A local symmetry only exists at a certain point, while a global one subsists at every point in the space-time. A continuous group of transformations under which a Lagrangian is invariant, is called a gauge group.

### A.1 Gauge Groups

From the various gauge groups existing in the different models of modern physics, we will focus on those used in this thesis. These are the 'unitary group'  $U(N)$  and the 'special unitary group'  $SU(N)$ . Unitary means e.g. for a matrix that its inverse is equal to its hermitian conjugated matrix  $U^{-1} = U^\dagger$ .

The unitary group  $U(N)$  - formerly hyperorthogonal group - is a real Lie-group of the dimension  $N^2$ . The dimension of a group determines the number of gauge bosons needed.  $U(N)$  consists of unitary  $N \times N$  matrices. We only use the simple case  $N = 1$ .  $U(1)$  is called the circle group since the group consists of all the complex numbers with an absolute value of 1 under matrix multiplication operations.

The special unitary group  $SU(N)$  is a subgroup of the unitary group  $U(N)$  and a Lie-group of the dimension  $N^2 - 1$ . The  $SU(N)$  consists of unitary  $N \times N$  matrices with complex entries and the Jacobian 1.



## A.2 Standard Model Gauge Group

The standard Model gauge group is  $G^{SM} = SU(3)_{QCD} \times SU(2)_I \times U(1)_Y$  [2]. It consists of the group  $SU(3)$  from quantum chromodynamics with eight gauge bosons (dimension of  $SU(3)$  is  $N^2 - 1 = 8$ ), the gluons. The group  $SU(2)_I \times U(1)_Y$  originates from the electroweak interaction. ‘I’ denotes the weak Isospin and ‘Y’ the hypercharge. Together they yield an additional four gauge bosons: The photon from  $U(1)_Y$  and the three (dimension of  $SU(2)$  is  $N^2 - 1 = 3$ ) weak interaction bosons  $Z, W^\pm$  from  $SU(2)_I$ .

## A.3 LR-symmetric Model Gauge Groups

The left-right symmetric model is an extension to the SM. In the low energy limit it becomes the SM. At high energies it breaks the SM symmetry from the electroweak interaction. This can be seen from its gauge group  $G^{LR} = SU(3)_{QCD} \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  with different couplings  $g_R \neq g_L$  for right- and left-handed particles.

## B MadGraph

MadGraph (MG) [17] is a program to generate matrix elements. It is usable directly on the homepage <http://madgraph.phys.ucl.ac.be/> or can be downloaded from the launchpad at <https://launchpad.net/madgraph5>. In both cases it is necessary to have certain programs installed or upgraded to the specific versions MG uses. A list of these depending on the operating system in use can be found on the tutorial pages e.g.:

<https://cp3.irmp.ucl.ac.be/projects/madgraph/wiki/SchoolNTU>.

### B.1 How to use MadGraph

From the directory where MG5 is saved, the program can be started by typing:

```
> ./bin/mg5
```

in a terminal. The startup output should look like this:

```
*****
*
*           W E L C O M E   t o   M A D G R A P H   5
*
*           *                               *
*           *           * *               *
*           * * * * 5 * * * *
*           *           * *               *
*           *                               *
*
*   VERSION 1.5.11                               2013-06-21
*
*   The MadGraph Development Team - Please visit us at
*   https://server06.fynu.ucl.ac.be/projects/madgraph
*
*           Type 'help' for in-line help.
*           Type 'tutorial' to learn how MG5 works
*
*****
```

```

load MG5 configuration from input/mg5_configuration.txt
Using default text editor "vi". Set another one in ./
    input/mg5_configuration.txt
Using default eps viewer "evince". Set another one in ./
    input/mg5_configuration.txt
Using default web browser "firefox". Set another one in
    ./input/mg5_configuration.txt
Checking if MG5 is up-to-date... (takes up to 2s)
No new version of MG5 available
Loading default model: sm
INFO: load particles
INFO: load vertices
INFO: Restrict model sm with file models/sm/
    restrict_default.dat .
INFO: Run "set stdout_level DEBUG" before import for
    more information.
INFO: Change particles name to pass to MG5 convention
Defined multiparticle p = g u c d s u~ c~ d~ s~
Defined multiparticle j = g u c d s u~ c~ d~ s~
Defined multiparticle l+ = e+ mu+
Defined multiparticle l- = e- mu-
Defined multiparticle vl = ve vm vt
Defined multiparticle vl~ = ve~ vm~ vt~
Defined multiparticle all = g u c d s u~ c~ d~ s~ a ve
    vm vt e- mu- ve~ vm~ vt~ e+ mu+ t b t~ b~ z w+ h w-
    ta- ta+

```

There are several models already implemented in MadGraph. These are all saved in the directory called “models” within the MadGraph5 directory. Every new model has to be saved in this directory as well. The TypeIII SeeSaw file is downloaded from <http://feynrules.irmp.ucl.ac.be/wiki/TypeIIISeeSaw>. From this file the model can be generated using FeynRules. Then the model can be saved in the “models” directory under a name e.g. typeIII. To import the new model type:

```
> import model typeIII
```

or

```
> import typeIII
```

Both commands import the model correctly. In the latter MG guesses that “typeIII” is a model, so it works only for unique names.

```

INFO: load particles
INFO: load vertices
INFO: Change particles name to pass to MG5 convention
impossible to set default multiparticles vl because No
particle ve in model
impossible to set default multiparticles vl~ because No
particle ve~ in model
Kept definitions of multiparticles l- / j / l+ / p
unchanged
Removed obsolete multiparticles all / vl / vl~
Defined multiparticle all = g a vl v2 v3 u c t d s b u~
c~ t~ d~ s~ b~ z w+ h w- tr0 e- mu- ta- tr- e+ mu+ ta
+ tr+

```

The MG output ‘impossible to set default multiparticles vl because No particle ve in model’ seems like an error message but its nothing to worry about. This just means that the new neutrino triplet is implemented with new particle (PDG) numbers, while the old neutrinos are deleted. Hence, MG does not find the neutrinos where it expects them to be and therefore deletes the obsolete (anti) neutrino.

To generate a simple process, just the three commands in the following are needed:

```

> generate p p > u d
> output NAME
> launch

```

The first command states which process you want to calculate. In the newer MadGraph versions, the particles have to be separated by a blank space. In the last output line above, all implemented particle names are listed. The sign “>” separates initial and final particles. One initial particle means a decay. With two initial particles MadGraph calculate the collision of these two. There are several commands to specify how exactly all branches of all decays should look like, but those will not be stated here.

The `output` command assigns the directory “NAME” to save the calculation there. The command `launch` initiates the actual calculation. The result is stated in a

crossx.html file in the “NAME” directory. If you open this file during the calculation, the progress is listed there.

## B.2 Example

In this section we demonstrate the usage of MadGraph with the example process  $u\bar{d} \rightarrow W^+$ . First we start MadGraph as shown in the section above. In the last lines of the startup MadGraph provides a list of the particle content. For the example process we type:

```
> generate u d > w+
```

MadGraph’s output should look like:

```
INFO: Checking for minimal orders which gives processes.
INFO: Please specify coupling orders to bypass this step
.
INFO: Trying process: u d~ > w+ WEIGHTED=2
INFO: Process has 1 diagrams
1 processes with 1 diagrams generated in 0.026 s
Total: 1 processes with 1 diagrams
```

Now we assign a directory to save and launch the process.

```
> output EXAMPLE
> launch
```

After the `launch` command MadGraph will request some parameters concerning the calculation. We just enter the default values, which let MG choose the appropriate options. While calculating MadGraph states the current process:

```
Generating 10000 events with run name run_01
survey run_01
compile directory
Using random number seed offset = 21
Running Survey
Creating Jobs
Working on SubProcesses
  PO_qq_wp
  Idle: 0 Running: 0 Finish: 1
End survey
```

```
refine 10000
Creating Jobs
Refine results to 10000
    PO_qq_wp
    Idle: 10 Running: 2 Finish: 0
    Idle: 7 Running: 2 Finish: 3
    Idle: 4 Running: 2 Finish: 6
    Idle: 1 Running: 2 Finish: 9
Combining runs
finish refine
refine 10000
Creating Jobs
Refine results to 10000
    PO_qq_wp
Combining runs
finish refine
combine_events
Combining Events
=== Results Summary for run: run_01 tag: tag_1 ===

    Cross-section :    3.959e+04 +- 53.97 pb
    Nb of events  :    10000

Creating Plots for parton level
End Plots for parton level
store_events
Storing parton level results
End Parton
quit

more information in /home/laura/MadGraph5/EXAMPLE/index.
html
```

Here it is stated that MadGraph uses 10000 events for the calculation. This is the default value and can be changed in the run card. The center of mass energy with the default value  $\sqrt{s} = 14$  TeV and collider mode pp collision are also stored in the run card. We will not change these values.

### B.3 Changes in MadGraph

If a zero cross section is encountered, this may be because of a zero coupling. It is possible to query interactions from MG:

```
> display interactions w- e+ tr0
```

Interactions 103 : e+ tr0 w- has the following property:

```
{
  'id': 103,
  'particles': [-11,8000018,-24],
  'color': [1 ],
  'lorentz': ['FFV2', 'FFV3'],
  'couplings': {(0, 1): 'GC_187', (0, 0): 'GC_171'},
  'orders': {'QED': 1}
}
```

Now we have to search for the couplings GC\_171 and GC\_187 in the file 'couplings.py' in the model directory. In the file we find:

```
GC_171 = Coupling(name = 'GC_171',\
                  value = '(ee*complex(0,1)*gCCL14)/(sw*cmath.sqrt(2))',
                  order = {'QED':1})
GC_187 = Coupling(name = 'GC_187',
                  value = '(ee*complex(0,1)*gCCR14)/sw',
                  order = {'QED':1})
```

We do not change anything here and continue with searching for gCCR14 and gCCL14. This factor can be found in the file 'parameters.py':

```
gCCL14 = Parameter(name = 'gCCL14',
                  nature = 'internal',
                  type = 'real',
                  value = '-Ve',
                  texname = '\\text{gCCL14}')
```

---

```
gCCR14 = Parameter(name = 'gCCR14',
                    nature = 'internal',
                    type = 'real',
                    value = '-((Ve*yme*math.sqrt(2))/mtr)',
                    texname = '\\text{gCCR14}')
```

Again we do not change anything here but search for `Ve`, `yme` and `mtr`. We find these also in the file 'parameter.py' in the model's directory.

```
Ve = Parameter(name = 'Ve',
                nature = 'external',
                type = 'real',
                value = 0,
                texname = '\\text{Ve}',
                lhablock = 'MIXING',
                lhacode = [ 1 ])
yme = Parameter(name = 'yme',
                 nature = 'external',
                 type = 'real',
                 value = 0,
                 texname = '\\text{yme}',
                 lhablock = 'YUKAWA',
                 lhacode = [ 13 ])
mtr = Parameter(name = 'mtr',
                 nature = 'external',
                 type = 'real',
                 value = 100.8,
                 texname = '\\text{mtr}',
                 lhablock = 'NEWMASSES',
                 lhacode = [ 4 ])
```

We will change these parameters. The values changed in the latter file are stated in the thesis.



## C Mathematica

Mathematica is a computational software tool developed by Wolfram Research. It can be obtained from their website <sup>2</sup>. We use the Martin-Stirling-Thorne-Watt Parton Distribution Functions [14]. All of its files, including an example, can be downloaded from this page <sup>3</sup>. The implemented pdfs have the form:

```
> xf[ih,x,q,f]
```

They return  $x$  times the momentum fraction  $x$  for a certain flavour  $f$ . The parameter  $ih$  denotes the set of eigenvectors and  $q$  the scale in GeV. According to the PDG convention for flavour,  $f=-1$  means  $\bar{d}$  and  $f=2$  means  $u$  quark. We use the central eigenvector set  $ih=0$  and a factorization scale of  $q=91$ , roughly equal to the  $Z$  mass. Using this we initialise the pdfs by typing:

```
> pdfu[x1_] := xf[0, x1, 91, 2]/x1
> pdfdbar[x1_] := xf[0, x1, 91, -1]/x1
```

We can now rewrite the underbraced part of equation (44)

$$\sigma (u\bar{d} \rightarrow W^+) = 0.105936 \int_0^1 dx_2 \underbrace{f_u \left( \frac{m_W^2}{x_2 s'} \right) f_{\bar{d}}(x_2)}_{\frac{1}{x_2 s'}}$$

into the command:

```
> pdf[x2_] := pdfu[mw^2/s/x2]*pdfdbar[x2]*1/x2*1/s /. params
```

The `/. params` part refers to a parameter array consisting of the known values we use. We integrate from the lower bound `lowerbound = mw^2/s /. params` using the command:

```
> integration = NIntegrate[pdf[x2], {x2, lowerbound, 1. },
MaxRecursion -> 100]
```

The command `NIntegrate` gives a numerical approximation. This approximated result multiplied with the constant factor from (44) and the factor from the unit conversion (45) yields: 40349 pb.

<sup>2</sup><http://www.wolfram.com/mathematica/>

<sup>3</sup><http://mstwpdf.hepforge.org/code/code.html>

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